Calculus for the Life Sciences I

Lecture Notes – Linear Models

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Outline

Snowy Tree Cricket

Chirping Crickets and Temperature

- Folk method for finding temperature (Fahrenheit)
  Count the number of chirps in a minute and divide by 4, then add 40
- In 1898, A. E. Dolbear [3] noted that
  "crickets in a field [chirp] synchronously, keeping time as if led by the wand of a conductor"

Chirping Crickets and Temperature

- Folk method for finding temperature (Fahrenheit)
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- In 1898, A. E. Dolbear [3] noted that
  "crickets in a field [chirp] synchronously, keeping time as if led by the wand of a conductor"
- He wrote down a formula in a scientific publication (first?)

$$T = 50 + \frac{N - 40}{4}$$

- Does this formula of Dolbear match the folk method described above?

[3] A. E. Dolbear, The cricket as a thermometer, American Naturalist (1897) 31, 970-971

Data Fitting Linear Model

- Mathematical models for chirping of snowy tree crickets, *Oecanthulus fultoni*, are **Linear Models**

[2] C. A. Bessey and E. A. Bessey, Further notes on thermometer crickets, American Naturalist (1898) 32, 263-264

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- Data from C. A. Bessey and E. A. Bessey [2] (8 crickets) from Lincoln, Nebraska during August and September, 1897 (shown on next slide)

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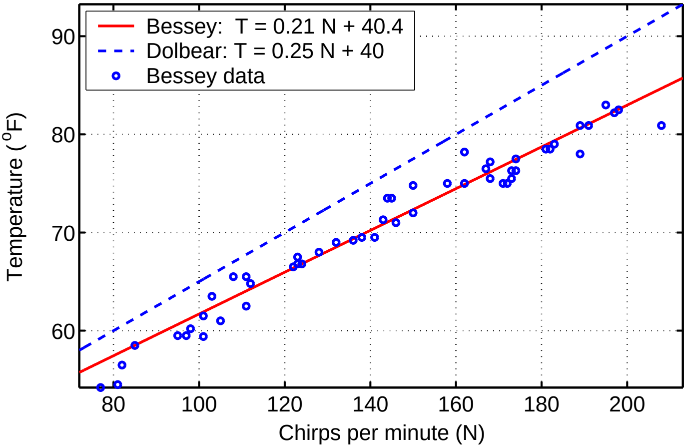
Data Fitting Linear Model

- Mathematical models for chirping of snowy tree crickets, *Oecanthulus fultoni*, are **Linear Models**
- Data from C. A. Bessey and E. A. Bessey [2] (8 crickets) from Lincoln, Nebraska during August and September, 1897 (shown on next slide)
- The least squares best fit line to the data is

$$T = 60 + \frac{N - 92}{4.7}$$

[2] C. A. Bessey and E. A. Bessey, Further notes on thermometer crickets, American Naturalist (1898) 32, 263-264

Bessey Data and Linear Models



Model

Cricket Equation as a Linear Model

The line creates a mathematical model

- The **temperature,** T as a **function** of the rate snowy tree crickets chirp, **Chirp Rate,** N

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There are several Biological and Mathematical questions about this **Linear Cricket Model**

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- The **temperature,** T as a **function** of the rate snowy tree crickets chirp, **Chirp Rate,** N

There are several Biological and Mathematical questions about this **Linear Cricket Model**

There is a complex relationship between the biology of the problem and the mathematical model

Biological Questions – Cricket Model 1

How well does the line fitting the Bessey & Bessey data agree with the Dolbear model given above?

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  - Different cricket species
  - Regional variation
  - Folk only an approximation

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- Graph shows Linear model fits the data well
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  - Different cricket species
  - Regional variation
  - Folk only an approximation
- Graph shows only a few °F difference between models

Biological Questions – Cricket Model 2

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When can this model be applied from a practical perspective?

- Biological thermometer has limited use
- Snowy tree crickets only chirp for a couple months of the year and mostly at night
- Temperature needs to be above 50°F

Mathematical Questions – Cricket Model 1

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- **Range** of **Linear functions** affects its **Domain**

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- Biologically, observations are mostly between 50°F and 85°F
- Thus, limited **range** of temperatures, so limited **range** on the **Linear Model**
- **Range** of **Linear functions** affects its **Domain**
- From the graph, **Domain** is approximately 50–200 **Chirps/min**

Mathematical Questions – Cricket Model 2

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- Observe that the temperature data trends lower at higher chirp rates – compared against linear model
- Better fit with **Quadratic function** – Is this really significant?

Equation of Line – Slope-Intercept Form

The **Slope-Intercept** form of the **Line**

$$y = m\,x + b$$

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The **Slope-Intercept** form of the **Line**

$$y = m\,x + b$$

- The variable x is the **independent variable**
- The variable y is the **dependent variable**
- The **slope** is m
- The y**-intercept** is b

Equation of Line – Cricket-Thermometer

The folk/Dolbear model for the cricket thermometer

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The folk/Dolbear model for the cricket thermometer

$$T = \frac{N}{4} + 40$$

- The independent variable is N, chirps/min
- The dependent variable is T, the temperature
- Thus, the temperature can be estimated from counting the number of chirps/min
- Equivalently, the temperature (measurement) depends on how rapidly the cricket is chirping

Equation of Line – Point-Slope Form

The **Point-Slope** form of the **Line** is often the most useful form

$$y - y_0 = m(x - x_0)$$

or

$$y = m(x - x_0) + y_0$$

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$$y - y_0 = m(x - x_0)$$

or

$$y = m(x - x_0) + y_0$$

- The **slope** is m
- The given **point** is (x_0, y_0)
- Again the independent variable is x, and the dependent variable is y

Equation of Line – Two Points

Given two points (x_0, y_0) and (x_1, y_1), the **slope** is given by

$$m = \frac{y_1 - y_0}{x_1 - x_0}$$

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Given two points (x_0, y_0) and (x_1, y_1), the **slope** is given by

$$m = \frac{y_1 - y_0}{x_1 - x_0}$$

Use the previous **point-slope** form of the line satisfies

$$y = m(x - x_0) + y_0$$

where the slope is calculated above and either point can be used.

Example – Slope and Point

Find the equation of a line with a slope of 2, passing through the point $(3, -2)$.
What is the y-intercept?

Skip Example

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The point-slope form of the equation gives:

$$\begin{aligned} y - (-2) &= 2(x - 3) \\ y + 2 &= 2\,x - 6 \\ y &= 2\,x - 8 \end{aligned}$$

Example – Two Points

Find the equation of a line passing through the points $(-2, 1)$ and $(3, -2)$

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From the point-slope form of the line equation, using the first point

$$\begin{aligned} y - 1 &= -\frac{3}{5}(x+2) \\ y &= -\frac{3}{5}x - \frac{1}{5} \end{aligned}$$

Parallel and Perpendicular Lines

Consider two lines given by the equations:

$$y = m_1x + b_1 \qquad \text{and} \qquad y = m_2x + b_2$$

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If $b_1 = b_2$, then the lines are the same.

The two lines are **perpendicular** if the slopes are negative reciprocals of each other, that is

$$m_1 m_2 = -1$$

Example – Perpendicular Lines 1

Find the equation of the line **perpendicular** to the line

$$5x + 3y = 6$$

passing through the point $(-2, 1)$

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Solution: The line can be written

$$\begin{aligned} 3y &= -5\,x + 6 \\ y &= -\frac{5}{3} + 2 \end{aligned}$$

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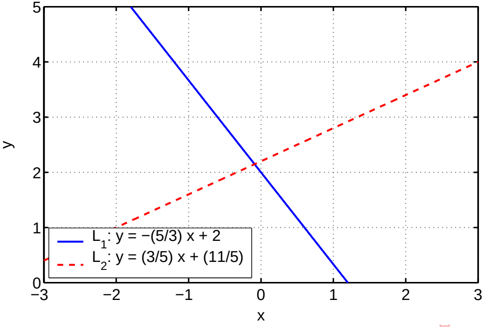
The slope of the perpendicular line (m_2) is the negative reciprocal

$$m_2 = \frac{3}{5}$$

Example – Perpendicular Lines 2

The point slope equation of the perpendicular line is

$$\begin{aligned} y - 1 &= \frac{3}{5}(x+2) \\ y &= \frac{3}{5}x + \frac{11}{5} \end{aligned}$$



Example – Intersection of Lines 1

Find the intersection of the line parallel to the line $y = 2x$ passing through $(1, -3)$ and the line given by the formula

$$3x + 2y = 5$$

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Solution: The line parallel to $y = 2x$ has slope $m = 2$, so satisfies

$$\begin{aligned} y + 3 &= 2(x - 1) \\ y &= 2x - 5 \end{aligned}$$

Example – Intersection of Lines 2

Continued: Substitute y into the formula for the second line

$$\begin{aligned} 3x + 2(2x - 5) &= 5 \\ 7x &= 15 \quad \text{or} \quad x = \frac{15}{7} \end{aligned}$$

Example – Intersection of Lines 2

Continued: Substitute y into the formula for the second line

$$\begin{aligned} 3x + 2(2x-5) &= 5 \\ 7x &= 15 \quad \text{or} \quad x = \frac{15}{7} \end{aligned}$$

Substituting the x value into the first line equation gives

$$y = 2\left(\frac{15}{7}\right) - 5 = -\frac{5}{7}$$

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The point of intersection is

$$(x, y) = \left(\frac{15}{7}, -\frac{5}{7}\right)$$

Metric System Conversion

All of the conversions for measurements, weights, temperatures, etc. are linear (or affine) relationships

Javascript Conversions

Example – Temperature 1

Convert Temperature Fahrenheit to Celsius

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- The freezing point of water is 32°F and 0°C, so take

$$(f_0, c_0) = (32, 0)$$

- The boiling point of water is 212°F and 100°C (at sea level), so take

$$(f_1, c_1) = (212, 100)$$

Example – Temperature 2

Convert Temperature Fahrenheit to Celsius

Solution: The slope satisfies

$$m = \frac{c_1 - c_0}{f_1 - f_0} = \frac{100 - 0}{212 - 32} = \frac{5}{9}$$

Example – Temperature 2

Convert Temperature Fahrenheit to Celsius

Solution: The slope satisfies

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The point-slope form of the line gives

$$\begin{aligned} c - 0 &= \frac{5}{9}(f - 32) \\ c &= \frac{5}{9}(f - 32) \end{aligned}$$

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The temperature f in Fahrenheit is the independent variable
The equation of the line gives the dependent variable c in Celsius

Example – Weight Conversion 1

Find the weight of a 175 pound man in kilograms.

Skip Example

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Solution: Tables show **1 kilogram** is **2.2046 pounds**

To convert pounds to kilograms, the slope for the conversion is

$$m = \frac{1 \text{ kg}}{2.2046 \text{ lb}} = 0.45360 \text{ kg/lb}$$

Example – Weight Conversion 2

Solution (cont): Let x be the weight in pounds and y be the weight in kilograms, then

$$y = 0.45360\,x$$

Example – Weight Conversion 2

Solution (cont): Let x be the weight in pounds and y be the weight in kilograms, then

$$y = 0.45360\,x$$

Thus, a 175 lb man is

$$y = 0.45360(175) = 79.38 \text{ kg}$$

Inverse Linear Function

Linear functions always have an Inverse

(Provided $m \neq 0$)

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Solving for x

$$\begin{aligned} mx &= y - b \\ x &= \frac{y-b}{m} \end{aligned}$$

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The **Inverse Line** satisfies

$$x = \left(\frac{1}{m}\right) y - \frac{b}{m}$$

Example - Inverse Line

The equation for converting °F to °C is

$$c = \frac{5}{9}(f - 32)$$

So,

$$f - 32 = \frac{9}{5}c$$

The equation for converting °C to °F is

$$f = \frac{9}{5}c + 32$$

Juvenile Height – Data

The table below gives the **average juvenile height** as a function of age [4]

Age (yr)	1	3	5	7	9	11	13
Height (cm)	75	92	108	121	130	142	155

Juvenile Height – Data

The table below gives the **average juvenile height** as a function of age [4]

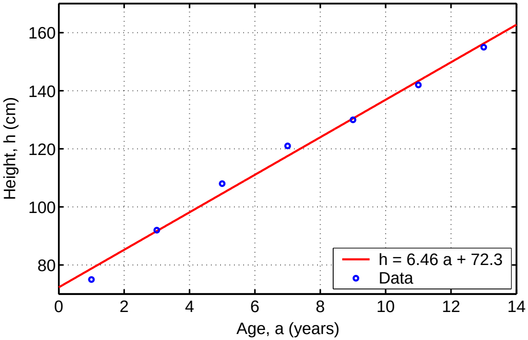
Age (yr)	1	3	5	7	9	11	13
Height (cm)	75	92	108	121	130	142	155

The data almost lie on a line, which suggests a **Linear Model**

[4] David N. Holvey, editor, The Merck Manual of Diagnosis and Therapy (1987) 15th ed., Merck Sharp & Dohme Research Laboratories, Rahway, NJ.

Juvenile Height – Graph

Below is a graph of the data and the besting fitting **Linear Model**



Juvenile Height – Linear Model

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The next section will explain finding the linear least squares best fit or linear regression to the data

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Model is valid for ages 1 to 13, the range of the data

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For modeling, it is valuable to place units on each part of the equation

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- Since the age, a, has units of years, the slope, m, has units of cm/year
- The slope is the rate of growth

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The model predicts that the **average newborn** will be **72.3 cm**

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What questions can you answer with this mathematical model?

What height does the model predict for a newborn baby?

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The model predicts that the **average newborn** will be **72.3 cm**

However, this is outside the range of the data, which makes its value more suspect

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Solution: Let $a = 8$, then

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What would give a better estimate? (Hint: Local Analysis)

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Solution: Average the data at ages 7 and 9

$$h_{ave}(8) = \frac{121 + 130}{2} = 125.5 \text{ cm}$$

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The data shows the **average 7 year old** is **121 cm**

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The model predicts the **average 7 year old** is **117.5 cm**
The data shows the **average 7 year old** is **121 cm**

Clearly, the first estimate is the best for this particular girl

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  - The model predicts the average 20 year old to be 201.5 cm

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- The domain of this function is restricted to some interval around $1 < a < 13$
  - The model predicts the average 20 year old to be 201.5 cm
- Local Analysis
  - Average 8 year old height better predicted from 7 and 9 year olds (125.5 cm)
  - Average newborn better estimated by data for 1 and 3 year old (66.5 cm)

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- Data show faster growth rates between 0 and 5 and again between 9 and 13
    - Growth spurts occur
    - Design a nonlinear model – Other functions

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- Growth rates for girls and boys differ – split the data according to sex
- Data show faster growth rates between 0 and 5 and again between 9 and 13
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  - Design a nonlinear model – Other functions
- In Math 122, we study growth models with differential equations

Sea Urchin Growth Model 1

Linear Models are reasonable for estimating growth over short time periods

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Consider a population of white sea urchins (*Lytechinus pictus*)

- Mean diameter of 28 mm on June 1 ($t = 0$)
- Mean diameter of 33 mm on July 1 ($t = 30$)

Estimate the mean diameter for the population of *Lytechinus pictus* on June 20 ($t = 19$), July 10 ($t = 39$), August 1 ($t = 61$), and August 15 ($t = 75$)

Which estimates do you trust more and why?

Skip Example

Sea Urchin Growth Model – Solution 2

Solution: The growth model desired has the form:

$$d = mt + d_0$$

where d is the mean diameter (mm) of the urchin and t is the number of days after June 1

Sea Urchin Growth Model – Solution 2

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The data give two points

$$(t_0, d_0) = (0, 28) \quad \text{and} \quad (t_1, d_1) = (30, 33)$$

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The slope is

$$m = \frac{33 - 38}{30 - 0} = \frac{1}{6} \text{ mm/day}$$

Sea Urchin Growth Model – Solution 3

Solution (cont): The growth model satisfies:

$$
\begin{aligned}
d - 28 &= \frac{1}{6}(t - 0) \\
d &= \frac{1}{6}t + 28
\end{aligned}
$$

- The d-intercept is the initial diameter measurement – 28 mm
- The slope is the growth rate – $\frac{1}{6}$ mm/day

Sea Urchin Growth Model – Solution 4

Solution (cont): Model Predictions:

$$d = \frac{1}{6}t + 28$$

June 20 – 31.2 mm (19, 31.2)
July 10 – 34.5 mm (39, 34.5)

August 1 – 38.2 mm (61, 38.2)
August 15 – 40.5 mm (75, 40.5)

Sea Urchin Growth Model – Solution 4

Solution (cont): Model Predictions:

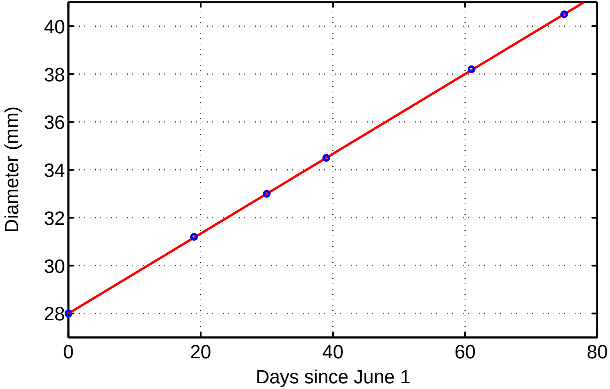
$$d = \frac{1}{6}t + 28$$

June 20 – 31.2 mm (19, 31.2)	August 1 – 38.2 mm (61, 38.2)
July 10 – 34.5 mm (39, 34.5)	August 15 – 40.5 mm (75, 40.5)

- The best estimate is for June 20 – it falls within data measurements
- The others are increasing more suspect
- Growth estimates are more accurate over shorter time intervals

Sea Urchin Growth Model – Graph

Below is a graph of the data and the **Linear Growth Model**



Scuba Diving Model 1

The pressure of air delivered by the regulator to a Scuba diver varies linearly with the depth of the water

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The regulator delivers air to a Scuba diver as follows:

- Air Pressure at 29.4 psi when at 33 ft
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Scuba Diving Model 1

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Find the pressure of air delivered at the surface (0 ft.), at 50 ft., and at 130 ft. (the maximum depth for recreational diving).

Scuba Diving Model – Solution 2

Solution: The linear model

$$p = md + p_0$$

where p is the pressure (psi) and d is the depth in feet

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Scuba Diving Model – Solution 2

Solution: The linear model

$$p = md + p_0$$

where p is the pressure (psi) and d is the depth in feet

The data give two points

$$(d_0, p_0) = (33, 29.4) \quad \text{and} \quad (d_1, p_1) = (66, 44.1)$$

The slope is

$$m = \frac{44.1 - 29.4}{66 - 33} = \frac{14.7}{33} = 0.445 \text{ psi/ft}$$

Scuba Diving Model – Solution 3

Solution (cont): The linear model satisfies

$$\begin{aligned} p - 29.4 &= 0.445(d - 33) \\ p &= 0.445\,d + 14.7 \end{aligned}$$

Scuba Diving Model – Solution 3

Solution (cont): The linear model satisfies

$$
\begin{aligned}
p - 29.4 &= 0.445(d - 33) \\
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\end{aligned}
$$

- At the surface, $d = 0$ and the air pressure is 14.7 psi
- At a depth of $d = 50$ ft, the air pressure is 36.95 psi
- At a depth of $d = 130$ ft, the air pressure is 72.55 psi

Scuba Diving Model – Solution 3

Solution (cont): The linear model satisfies

$$\begin{aligned} p - 29.4 &= 0.445(d - 33) \\ p &= 0.445\, d + 14.7 \end{aligned}$$

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- At a depth of $d = 50$ ft, the air pressure is 36.95 psi
- At a depth of $d = 130$ ft, the air pressure is 72.55 psi
- Note these assume we are at sea level and diving in sea water

Scuba Diving Model – Graph

Below is a graph of the data and the **Linear Pressure Model**