Calculus for the Life Sciences I

Lecture Notes – Introduction

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Outline

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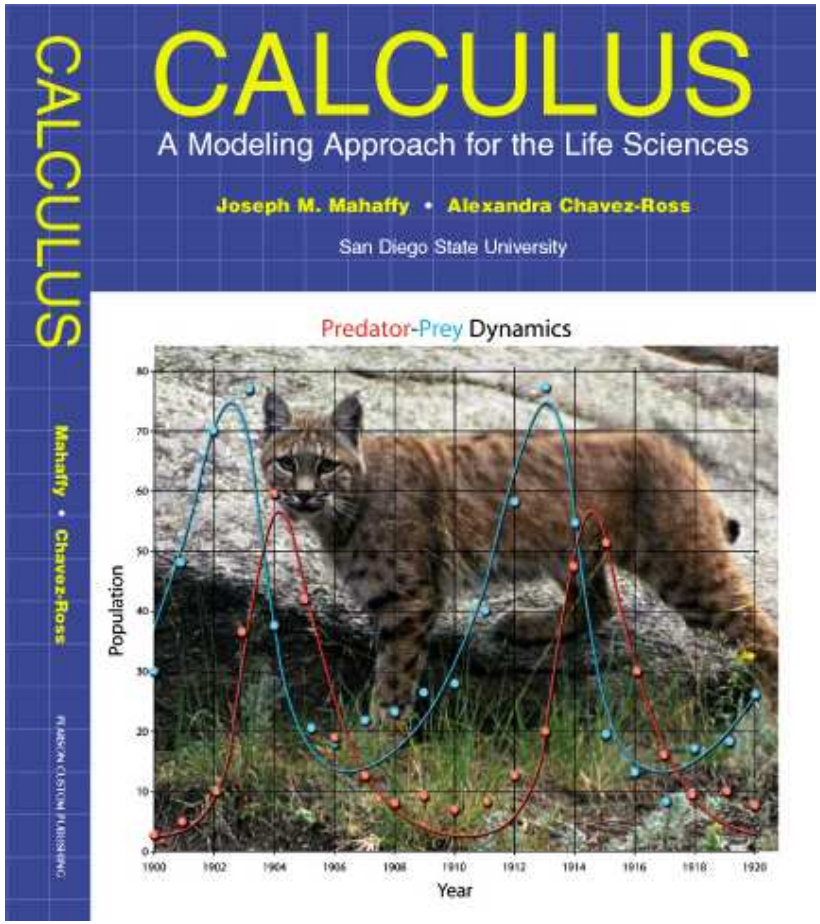
Email	**berardi@rohan.sdsu.edu**
Office Hours	12:15-1:45 W 1:30-3 Th in GMCS 425, and by appointment
TA	Nancy Tafolla
Email	**tafolla@rohan.sdsu.edu**
Office Hours	12-1:30 TTh in GMCS 425, and by appointment

Basic Information: The Book



Title:

"Calculus: A Modeling Approach for the Life Sciences"

8th Edition

Authors:

Joseph M. Mahaffy &
Alexandra Chàvez-Ross

Publisher:

Pearson Custom Publishing

ISBN:

0-558-17036-6

Basic Information: Syllabus

- Functions and Models
  - Linear Models
  - Least Squares Analysis
  - Quadratic and Other Functions
  - Allometric Modeling, Exponentials, Logarithms
- Discrete Dynamical Models
  - Malthusian Growth
  - Linear Discrete Models
- The Derivative
  - Basic Rules and Applications
  - Derivatives of Special Functions
  - Product Rule and Quotient Rule
  - Chain Rule
  - Optimization

Basic Information: Grading

Detailed information is found on the
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- Lecture Material is 2/3 of grade
  - Homework with WeBWorK (20% of Lecture grade)
  - 3 Exams (16% each)
  - Final (32%)
  - Scientific Calculator only - Exams and Final
  - One 3x5 notecard for Exams and three 3x5 notecards for Final

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- Lab Work is 1/3 of grade
  - 9-11 Lab assignments
  - 3 Lab Exams worth twice a regular Lab assignment
  - Open notes, Computer (except email)

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  - Abide by university statutes, and all applicable local, state, and federal laws.

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- Students are expected ***and encouraged*** to to make use of office hours! If you cannot make it to the scheduled office hours: contact the instructor to schedule an appointment!

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- **Lab assignments**:
  - Attendance is mandatory or automatic 10 point deduction
  - Partners are assigned and must work with given partner
  - Arriving 20 minutes late or missing a Lab means working the lab alone
  - Labs due promptly by Thursday 9 PM following a given Lab unless told otherwise.
  - Lowest lab score is dropped
  - Your responsibility to back up Lab work – No excuses accepted or extensions granted for lost material

Computer Lab

- Computer Labs are located in GMCS 422 and 425 – Hours are posted on the Lab doors
- Completed Lab Reports are turned into Math 121 box located in GMCS 425
- Software used
  - Excel
  - Word
  - Maple
- Labs are 60% WeBWorK and 40% written report
- **Please direct questions first to your Lab TA**

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  - Begin with a biological model
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  - Mathematical theory required to analyze the biological problem
- Use real or realistic examples
- Computer labs aid the more complicated models

Math 121: Introduction — Mathematical Biology

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- Mathematical tools
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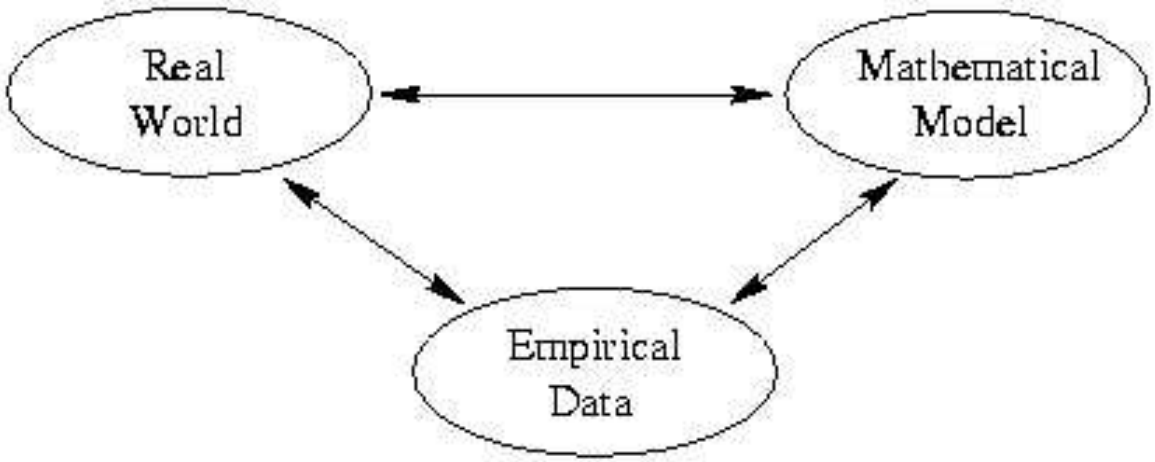
- Biological problems
  - Often stretch mathematical techniques
  - Illustrate mathematical tools well
  - Build intuition for problem techniques

Math 121: Introduction — Mathematical Model

So what is a mathematical model?



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- The model may suggest improved experiments
- Often there is not an exact answer, differing from K-12 training in mathematics

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- Adult onset diabetes (Type II) results in insulin resistance – cells fail to use insulin properly

Diabetes mellitus – Ackerman Model

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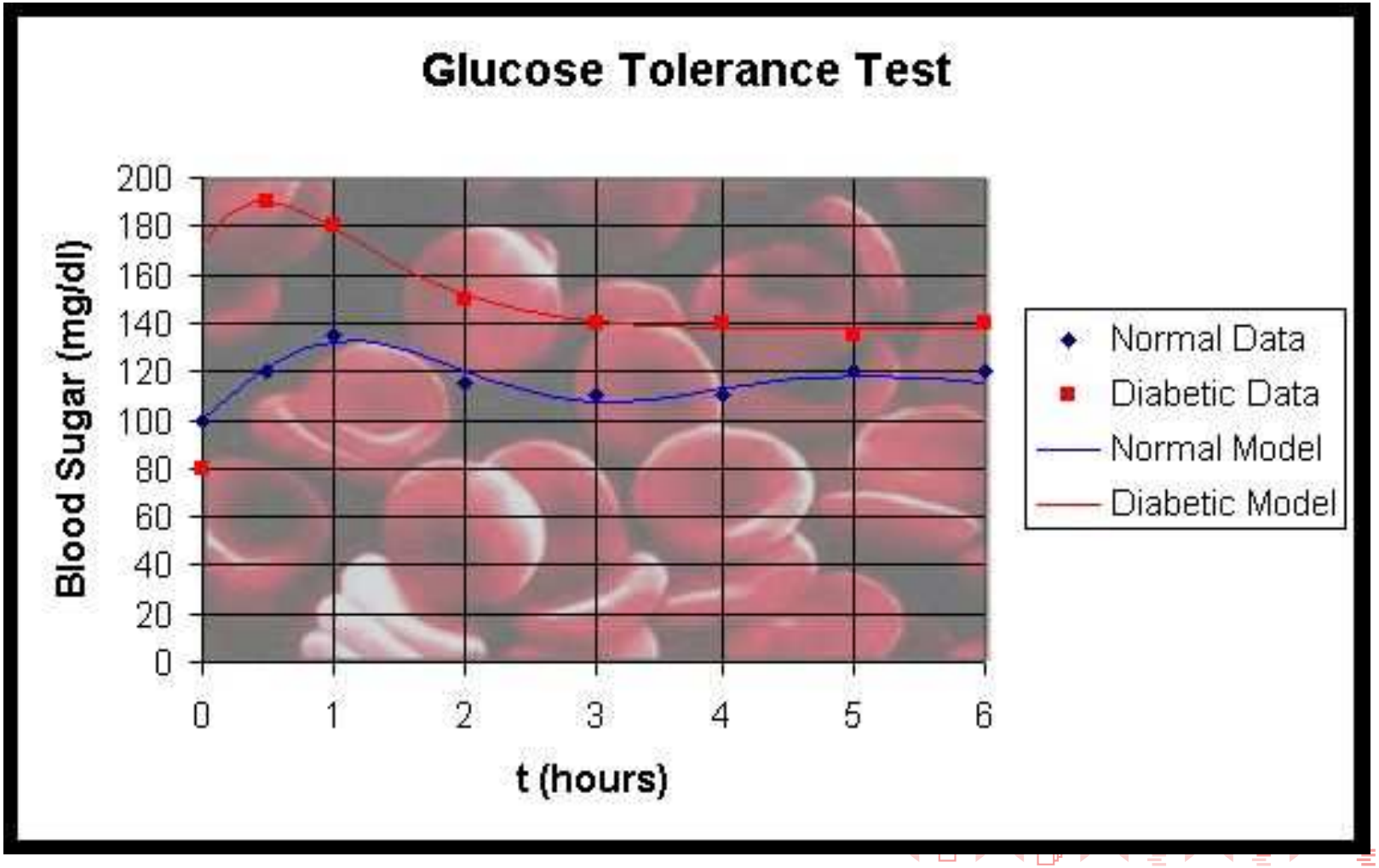
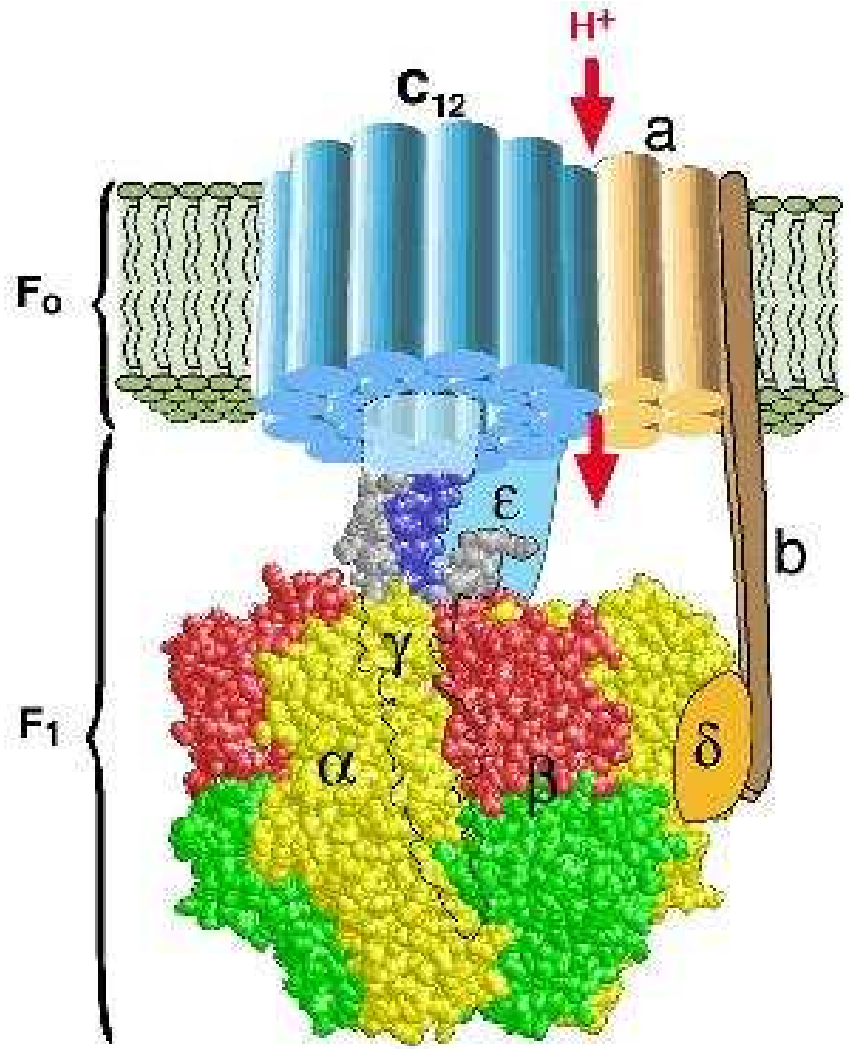
- Mathematical Model
  - 2-Component model - Blood glucose and insulin levels
  - Linear system of differential equations (Damped harmonic oscillator)
  - Simple solution with exponentials and trig functions
  - Solution fit to data
  - Parameter values indicate health of subject

Ackerman Model for Diabetes

Glucose Tolerance Test



Introduction – Example 2 – ATP synthase



H. Wang and G. Oster (1998). Nature 396:279-282.

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- The 90+% efficiency of this molecule cannot be explained by physical laws of thermodynamics for cleaving (or forming) this phosphate from ATP

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  Brownian rachet diagram for ATP synthase
- Nobel prize in 1997 for Chemistry was awarded to Paul D. Boyer, John E. Walker, and Jens C. Skou for some of the work