Calculus for the Life Sciences I

Lecture Notes – Introduction

Joseph M. Mahaffy,
⟨mahaffy@math.sdsu.edu⟩

Department of Mathematics and Statistics
Dynamical Systems Group
Computational Sciences Research Center
San Diego State University
San Diego, CA 92182-7720

http://www-rohan.sdsu.edu/~jmahaffy

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Outline

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Contact Information

Office	GMCS-593
Email	jmahaffy@mail.sdsu.edu
Web	http://www-rohan.sdsu.edu/~jmahaffy
Phone	(619)594-3743
Office Hours	1-2 MW and 3-4 MW, and by appointment

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TA Contact Information

TA	Vinnie Berardi
Email	berardi@rohan.sdsu.edu
Office Hours	12:15-1:45 W 1:30-3 Th in GMCS 425, and by appointment
TA	Nancy Tafolla
Email	tafolla@rohan.sdsu.edu
Office Hours	12-1:30 TTh in GMCS 425, and by appointment

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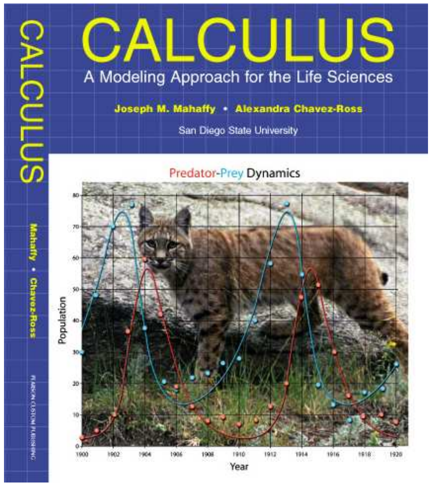
Basic Information: The Book



Title:
"Calculus: A Modeling Approach for the Life Sciences"
8th Edition

Authors:
Joseph M. Mahaffy &
Alexandra Chàvez-Ross

Publisher:
Pearson Custom Publishing

ISBN:
0-558-17036-6

Basic Information: Syllabus

- Functions and Models
  - Linear Models
  - Least Squares Analysis
  - Quadratic and Other Functions
  - Allometric Modeling, Exponentials, Logarithms
- Discrete Dynamical Models
  - Malthusian Growth
  - Linear Discrete Models
- The Derivative
  - Basic Rules and Applications
  - Derivatives of Special Functions
  - Product Rule and Quotient Rule
  - Chain Rule
  - Optimization

Basic Information: Grading

Detailed information is found on the
Homework and Assignment Web Page

- Lecture Material is 2/3 of grade
  - Homework with WeBWorK (20% of Lecture grade)
  - 3 Exams (16% each)
  - Final (32%)
  - Scientific Calculator only - Exams and Final
  - One 3x5 notecard for Exams and three 3x5 notecards for Final
- Lab Work is 1/3 of grade
  - 9-11 Lab assignments
  - 3 Lab Exams worth twice a regular Lab assignment
  - Open notes, Computer (except email)

Expectations and Procedures, I

- Most lecture class attendance is OPTIONAL — Homework and announcements will be posted on the class web page. If/when you attend class:
  - Please be on time.
  - Please pay attention.
  - Please turn off mobile phones.
  - Please be courteous to other students and the instructor.
  - Abide by university statutes, and all applicable local, state, and federal laws.

Expectations and Procedures, II

- WeBWorK assignments are posted with a specific due date. It is **your responsibility** to complete the assignment on time.
- The instructor will make special arrangements for students with documented learning disabilities and will `try` to make accommodations for other unforeseen circumstances, *e.g.* illness, personal/family crises, etc. in a way that is fair to all students enrolled in the class. ***Please contact the instructor EARLY regarding special circumstances.***
- Students are expected ***and encouraged*** to ask questions in class!
- Students are expected ***and encouraged*** to to make use of office hours! If you cannot make it to the scheduled office hours: contact the instructor to schedule an appointment!

Expectations and Procedures, III

- **Missed Exams or Lab Exams: Don't miss Exams!** You will receive a **ZERO** for any missed exam, except for **written/documented** excuses (illness, personal/family crises, etc.).
- **Lab assignments**:
  - Attendance is mandatory or automatic 10 point deduction
  - Partners are assigned and must work with given partner
  - Arriving 20 minutes late or missing a Lab means working the lab alone
  - Labs due promptly by Thursday 9 PM following a given Lab unless told otherwise.
  - Lowest lab score is dropped
  - Your responsibility to back up Lab work – No excuses accepted or extensions granted for lost material

Computer Lab

- Computer Labs are located in GMCS 422 and 425 – Hours are posted on the Lab doors
- Completed Lab Reports are turned into Math 121 box located in GMCS 425
- Software used
  - Excel
  - Word
  - Maple
- Labs are 60% WeBWorK and 40% written report
- **Please direct questions first to your Lab TA**

Math 121: Formal Prerequisites

- Successful Completion of ELM Exam
- Good knowledge of High School Algebra
- Reasonable score on Algebra Self-Test - **WeBWorK**
  - Take as an **Exam** - Don't ask others or check notes
  - Scientific Calculator only
  - Time for **2 hours**
  - Score at least **70%**
  - Review missed questions and correct

Math 121: Introduction

- Biology is rapidly expanding - more quantitative analysis of the data
- Mathematics and computers are more important
- This course in Calculus for Biology
  - Emphasis on mathematical modeling of biological systems
  - Lecture notes show how Calculus naturally arises in biological examples
  - Begin with a biological model
  - Mathematical theory required to analyze the biological problem
- Use real or realistic examples
- Computer labs aid the more complicated models

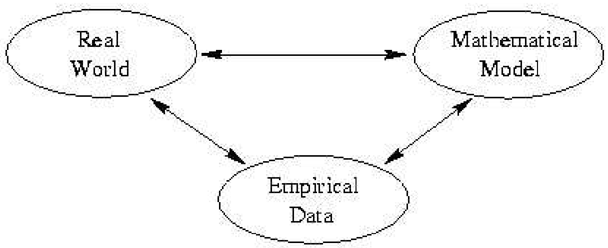
Math 121: Introduction — Mathematical Biology

Mathematical Biology

- Mathematical tools
  - Better qualitative and quantitative understanding of biological problems
  - Suggest alternate possibilities
  - Reject inconsistent ideas
- Biological problems
  - Often stretch mathematical techniques
  - Illustrate mathematical tools well
  - Build intuition for problem techniques

Math 121: Introduction — Mathematical Model

So what is a mathematical model?



Math 121: Introduction — Mathematical Model

- A **mathematical model** is a representation of a real system
- It is simple in design
- It exhibits the basic properties of the real system
- The model should be testable against empirical data
- Comparisons of the model to the real system should lead to improved mathematical models
- The model may suggest improved experiments
- Often there is not an exact answer, differing from K-12 training in mathematics

Introduction – Example – Diabetes mellitus

Biological Information

- Metabolic disease characterized by too much sugar in the blood and urine
- β-cells in the pancreas release insulin in response to rises in levels of glucose in the blood
- Stores energy as glycogen in the liver
- Juvenile diabetes (Type I) - failure of the β-cells to release insulin to blood glucose levels – probably an autoimmune response killing β-cells
- Adult onset diabetes (Type II) results in insulin resistance – cells fail to use insulin properly

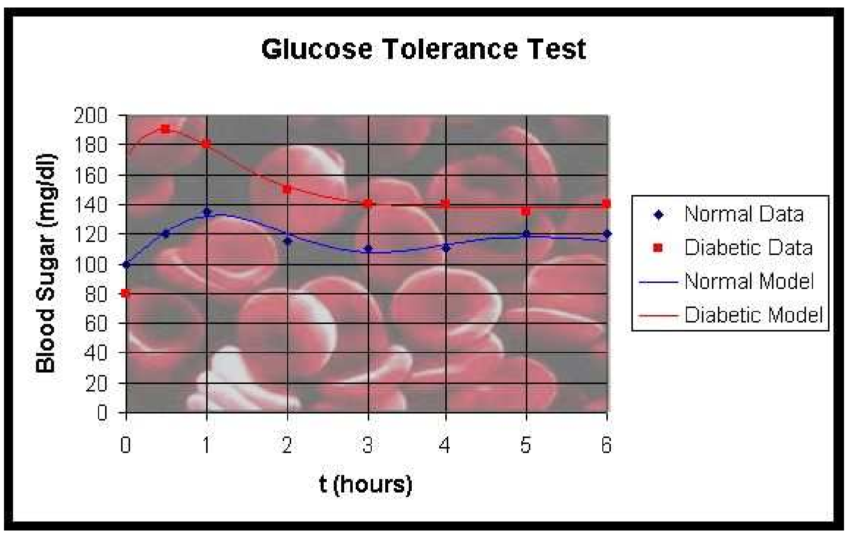
Diabetes mellitus – Ackerman Model

Ackerman Model for Diabetes

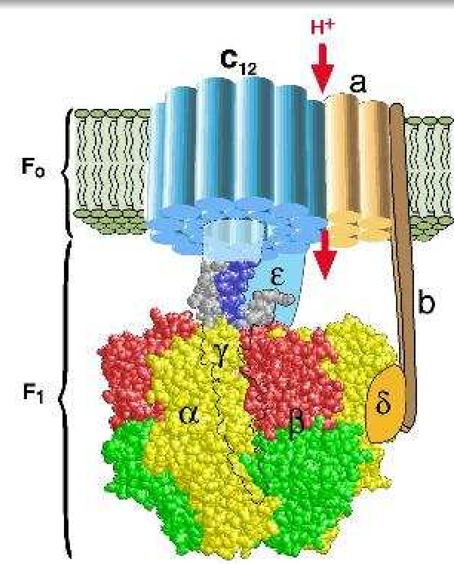
- Glucose Tolerance Test (GTT)
  - Subject fasts for 12 hours
  - Given a large quantity of glucose
  - Blood sampled regularly for 4-6 hours
- Mathematical Model
  - 2-Component model - Blood glucose and insulin levels
  - Linear system of differential equations (Damped harmonic oscillator)
  - Simple solution with exponentials and trig functions
  - Solution fit to data
  - Parameter values indicate health of subject

Ackerman Model for Diabetes

Glucose Tolerance Test



Introduction – Example 2 – ATP synthase



H. Wang and G. Oster (1998). Nature 396:279-282.

ATP synthase – Biological Information

- One of the most important molecules in all living organisms
- Store chemical energy in two forms
  - Transmembrane electrochemical gradients
  - Chemical bonds, particularly adenosine triphosphate (ATP)
- Optimized by evolution - few variations in its structure for all living organisms
- Standard texts state that energy is released by breaking the high energy gamma phosphate bond in ATP as a single event
- The 90+% efficiency of this molecule cannot be explained by physical laws of thermodynamics for cleaving (or forming) this phosphate from ATP

ATP synthase – Modeling

- Collaboration of many scientist from many fields, including some applied mathematicians, have elucidated the details
- The phosphate bond is formed in a series of 15 to 20 smaller steps like a zipper
- Energy gained against chemical gradient by a **Brownian rachet**
  Brownian rachet diagram for ATP synthase
- Nobel prize in 1997 for Chemistry was awarded to Paul D. Boyer, John E. Walker, and Jens C. Skou for some of the work