1. a. The best allometric model for the food consumption F as a function of the weight w is given by:

$$F = 34.601w^{0.6577}.$$

This is consistent with most energy generated by food being lost through the surface of an animal.

b. Below is a table with the model predictions of the food consumption and the percent error for each dog:

Dog	Weight (kg)	Food (g)	Model Food	% Error
Fox Terrier	4	85	86.11	1.308
Dachshund	12	180	177.36	-1.464
Brittany Spaniel	17	220	223.03	1.375
Dalmatian	27	310	302.34	-2.471
Labrador Retriever	33	360	345.00	-4.168
Bloodhound	41	380	397.94	4.721
Great Dane	68	550	555.05	0.918

From the error analysis, we see that there is fairly uniform percent error, so any of the dogs is equally likely to have the highest error from the model. The bloodhound shows the highest percent error, which could be due to low activity (food consumption below predicted value). The dog with the lowest percent error is the Great Dane, which could be because it is much larger than the others and has greater weight affecting the model.

c. With the model, we fill in the table below.

Dog	Weight (kg)	Food (g)
Beagle	13	**186.95**
Golden Retriever	**32.28**	340
Rottweiler	53	**471.13**
Siberian Husky	**20.22**	250

d. Using the power rule (or Maple), we find the derivative of the allometric model is given by:

$$F'(w) = 22.757w^{-0.3423}.$$

From this formula, it easily follows that:

$$F'(20) = 22.757(20^{-0.3423}) = 8.1616 \qquad \text{and} \qquad F'(40) = 6.4377.$$

The derivative shows that with increasing size, the dogs utilize energy more efficiently. This is because larger dogs have a smaller surface area to volume ratio, and much of mammalian energy is consumed creating heat, which is lost through the surface.

2. a. The model given by

$$c(t) = c_\infty \left(1 - A\, e^{-bt}\right),$$

has the best fitting parameters $c_\infty = 2.52925$, $A = 0.998887$, and $b = 1.3125$. The sum of square errors is 0.0024580. The model satisfies $c(2) = 2.34624$ with a percent error of 1.569%. It satisfies $c(5) = 2.52568$ with a percent error of -0.9536%. The c-intercept is $c(0) = 0.00281484$. The horizontal asymptote matches $c_\infty = 2.52925$.

b. The model given by

$$C_{n+1} = \alpha C_n + \mu,$$

has the best fitting initial value $C_0 = 0.00281484$, while the best fitting parameters are $\alpha = 0.269147$ and $\mu = 1.8485$. The sum of square errors is the same at 0.0024580. The model is effectively the same, so satisfies $c(2) = 2.34624$ with a percent error of 1.569%. It satisfies $c(5) = 2.52568$ with a percent error of -0.9536%. The equilibrium for this model is $C_e = 2.52925$. It follows that these two models are mathematically equivalent.

c. From Maple, $c'(t) = 3.315946\, e^{-1.3125\, t}$. From this formula, it follows that $c'(0) = 3.315946$, $c'(1) = 0.89247$, and $c'(2) = 0.24021$. With the second model's values at $C_0 = 0.00281484$ and $C_2 = 2.34624$, it follows that

$$\Delta C_1 = \frac{C_2 - C_0}{2} = 1.171712580.$$

This value is a bit higher than $c'(1)$.