1. a. Write $f(x)$ as powers of x as much as possible (remove denominators), so

$$f(x) = 6x^3 + 2x^{-2} - e^{2x}(x^2 - 9).$$

Apply power rules, product rule, and the rules for exponential yielding

$$\begin{aligned} f'(x) &= 6(3x^2) + 2(-2x^{-3}) - \left(e^{2x}(2x) + 2e^{2x}(x^2 - 9)\right) \\ &= 18x^2 - \frac{4}{x^3} - 2e^{2x}(x^2 + x - 9) \end{aligned}$$

b. Use the properties of logarithms to write

$$g(x) = 2e^{-3x} + 2\ln(x) - 5.$$

Use the rules of differentiation of exponentials and logarithms to give

$$\begin{aligned} g'(x) &= 2(-3)e^{-3x} + \frac{2}{x} + 0 \\ &= \frac{2}{x} - 6e^{-3x} \end{aligned}$$

c. Leave $h(x)$ in the form,

$$h(x) = 2x^6 \ln(x) - e^{x^2+4x} + \frac{1}{2}e^{-4x}.$$

Apply power rules, product rule, chain rule, and the rules for exponentials and logarithms yielding

$$\begin{aligned} h'(x) &= 2\left((6x^5)\ln(x) + x^6\left(\frac{1}{x}\right)\right) - e^{x^2+4x}(2x+4) + \frac{-4}{2}e^{-4x} \\ &= 12x^5\ln(x) + 2x^5 - (2x+4)e^{x^2+4x} - 2e^{-4x} \end{aligned}$$

d. Write $k(t)$ as powers of t as much as possible, so

$$k(t) = \frac{1}{4}t^2 - 4t^{-\frac{1}{2}} + \frac{2 + e^{2t}}{t^2 - 3}.$$

Apply power rules, the rules for logarithms, and quotient rule yielding

$$\begin{aligned} k'(t) &= \left(\frac{2}{4}\right)t - 4\left(-\frac{1}{2}\right)t^{-\frac{3}{2}} + \frac{2(t^2-3)e^{2t} - 2t(2+e^{2t})}{(t^2-3)^2} \\ &= \frac{t}{2} + \frac{2}{\sqrt{t^3}} + \frac{2(t^2-3)e^{2t} - 2t(2+e^{2t})}{(t^2-3)^2} \end{aligned}$$

e. Write $p(w)$ as powers of w, so

$$p(w) = \frac{2\ln(w) + w}{w^3 - 8} - w^{2/5} + w^{-3}e^{-w}.$$

Apply power rules, product rule, and the rules for exponentials to give

$$\begin{aligned} p'(w) &= \frac{(w^3-8)(2/w+1)-3w^2(2\ln(w)+w)}{(w^3-8)^2} - \frac{2}{5}w^{-3/5} + \left(w^{-3}(-e^{-w}) - 3w^{-4}e^{-w}\right) \\ &= \frac{(w^3-8)(2/w+1)-3w^2(2\ln(w)+w)}{(w^3-8)^2} - \frac{2}{5}w^{-3/5} - \frac{e^{-w}}{w^3}\left(1+\frac{3}{w}\right) \end{aligned}$$

f. Write $q(z)$ as powers of z and use properties of logarithms, so

$$q(z) = 2Az\ln(z) - Bz^{1/2} + Cz^{-3}.$$

Apply power rules and the rules for logarithms to give

$$\begin{aligned} q'(z) &= 2A\left(\frac{z}{z}+\ln(z)\right) - \frac{B}{2}z^{-1/2} + C(-3)z^{-4} \\ &= 2A(1+\ln(z)) - \frac{B}{2\sqrt{z}} - \frac{3C}{z^4} \end{aligned}$$

g. Write $r(x)$ as follows:

$$r(x) = e^{2x}(x^3-5x+7)^4 - \frac{7x}{(x^2+2x+5)^{1/2}}.$$

Apply the product, quotient, and chain rule to obtain

$$\begin{aligned} r'(x) &= \left(e^{2x}4(x^3-5x+7)^3(3x^2-5) + 2e^{2x}(x^3-5x+7)^4\right) \\ &\quad - \frac{7(x^2+2x+5)^{1/2} - (7x/2)(x^2+2x+5)^{-1/2}(2x+2)}{(x^2+2x+5)} \\ &= 2e^{2x}(x^3-5x+7)^3(x^3+6x^2-5x-3) - \frac{7(x+5)}{(x^2+2x+5)^{3/2}} \end{aligned}$$

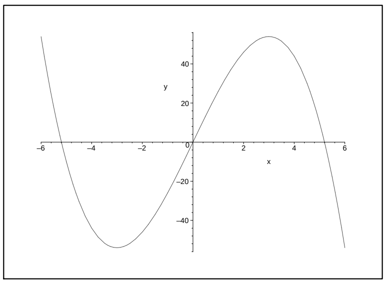
h. Leave $F(y)$ as

$$F(y) = (3y^2-4y+6)^5 + \ln(2y+9).$$

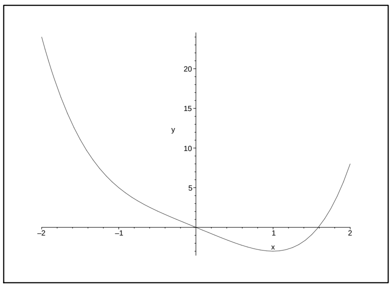
Apply the chain rule to obtain

$$F'(y) = 5(3y^2-4y+6)^4(6y-4) + \frac{2}{2y+9}$$

2. a. $y = 27x - x^3$
Domain is all x.
y-intercept: $y(0) = 0$, so $(0,0)$.
x-intercepts: $27x - x^3 = x(27-x^2) = 0$, so $x = 0$ and $x = \pm\sqrt{27} = \pm 3\sqrt{3}$.
No asymptotes
Derivative $y'(x) = 27 - 3x^2$
Extrema are where $y'(x) = -3(x^2-9) = 0$, so $x = \pm 3$. With $y(-3) = 27(-3) - (-3)^3 = -54$ and



Problem 2a

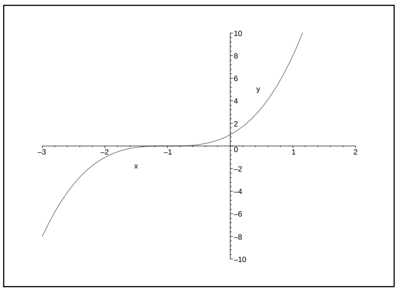


Problem 2b

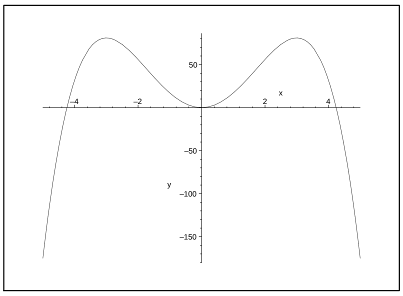
$y(3) = 54$. Thus, $(3, 54)$ is a maximum, and $(-3, -54)$ is a minimum.
Second derivative $y''(x) = -3(2)x = -6x$.
Point of inflection ($y'' = 0$): At $x = 0$ or $(0, 0)$.

b. $y = x^4 - 4x$
Domain is all x.
y-intercept: $y(0) = 0$, so $(0, 0)$.
x-intercept: $x^4 - 4x = x(x^3 - 4) = 0$, so $x = 0$ and $x = \sqrt[3]{4} \simeq 1.587$.
No asymptotes
Derivative $y'(x) = 4x^3 - 4$
Extrema are where $y'(x) = 4(x^3 - 1) = 0$, so $x = 1$. With $y(1) = 1^4 - 4(1) = -3$, $(1, -3)$ is a minimum.
Second derivative $y''(x) = 12x^2$.
Point of inflection ($y'' = 0$): At $x = 0$ or $(0, 0)$.

c. $y = x^3 + 3x^2 + 3x + 1$
Domain is all x.
y-intercept: $y(0) = 1$, so $(0, 1)$.
x-intercept: $x^3 + 3x^2 + 3x + 1 = (x + 1)^3 = 0$, so $x = -1$.
No asymptotes
Derivative $y'(x) = 3x^2 + 3(2)x + 3 = 3x^2 + 6x + 3$
Extrema are where $y'(x) = 3(x + 1)^2 = 0$, so $x = -1$ is a critical point. $y(-1) = 0$, but $(-1, 0)$ is a saddle point (neither maximum or minimum.
Second derivative $y''(x) = 3(2)x + 6 = 6x + 6$.
Point of inflection ($y'' = 0$): At $x = -1$ or $(-1, 0)$.



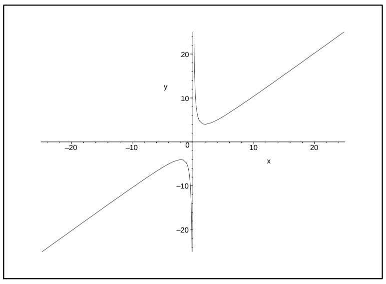
Problem 2c



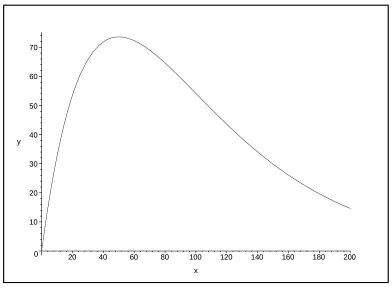
Problem 2d

d. $y = 18x^2 - x^4$
Domain is all x.
y-intercept: $y(0) = 0$, so $(0, 0)$.
x-intercept: $x^2(18 - x^2) = -x^2(x + 3\sqrt{2})(x - 3\sqrt{2}) = 0$, so $x = 0$ and $x = \pm 3\sqrt{2}$.
No asymptotes
Derivative $y'(x) = 36x - 4x^3 = 4x(9 - x^2)$
Critical points satisfy $y'(x) = -4x(x^2 - 9) = 0$, so $x = 0, \pm 3$. With $y(0) = 0$, $(0, 0)$ is a minimum.
When $x = \pm 3$,$y(\pm 3) = 81$, so there are local maxima at $(-3, 81)$ and $(3, 81)$.
Second derivative $y''(x) = 36 - 12x^2 = 12(3 - x^2)$.
Point of inflection ($y'' = 0$): At $x = \pm\sqrt{3}$, giving $(\pm\sqrt{3}, 45)$.

e. $y = x + \dfrac{4}{x} = x + 4x^{-1}$
Domain is all $x \neq 0$.
Since there is a vertical asymptote at $x = 0$, there is no y-intercept.
We solve $y = \frac{x^2+4}{x} = 0$ or $x^2 + 4 = 0$, so no x-intercepts.
Derivative $y'(x) = 1 - 4x^{-2} = \frac{x^2-4}{x^2}$
Critical points satisfy $y'(x) = 0$, so $x^2 - 4 = 0$ or $x = \pm 2$. With $y(-2) = -4$, $(-2, -4)$ is a local maximum. With $y(2) = 4$, $(2, 4)$ is a local minimum.
Second derivative $y''(x) = 8x^{-3}$, which is never zero, so no points of inflection.



Problem 2e

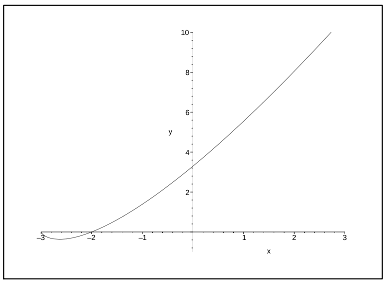


Problem 2f

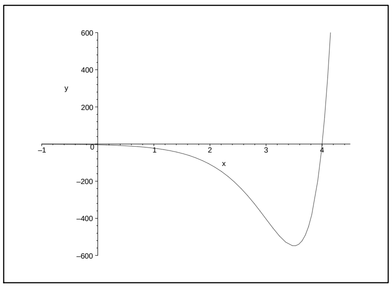
f. $y = 4xe^{-0.02x}$
Domain is all x.
y-intercept: $y(0) = 0$, so $(0, 0)$, which is also, the only x-intercept.
Horizontal asymptote: As $x \to \infty$, $y \to 0$, so $y = 0$ is a horizontal asymptote (looking to the right).
Derivative: By the product rule, $y'(x) = 4x(-0.02)e^{-0.02x} + 4e^{-0.02x} = 4e^{-0.02x}(1 - 0.02x)$
Critical points satisfy $y'(x) = 0$, so $1 - 0.02x = 0$ or $x = 50$. With $y(50) = 200e^{-1} \simeq 73.576$, $(50, 73.576)$ is a maximum.
Second derivative $y''(x) = 4e^{-0.02x}(-0.02) + 4(-0.02)e^{-0.02x}(1 - 0.02x) = -0.16(1 - 0.01x)e^{-0.02x}$.
Point of inflection ($y'' = 0$): At $x = 100$, $y(100) = 400e^{-2} \simeq 54.134$. Thus, $(100, 54.134)$.

g. $y = (x + 3)\ln(x + 3)$
Domain is $x > -3$. The y-intercept is $3\ln(3) \simeq 3.2958$.
x-intercept: Where $(x + 3)\ln(x + 3) = 0$, which occurs when $\ln(x + 3) = 0$ or $x = -2$.
There are no asymptotes. (It can be shown that as $x \to -3$, $y \to 0$.)
Derivative: By the product rule, $y'(x) = \frac{x+3}{x+3} + \ln(x + 3) = 1 + \ln(x + 3)$.

Critical points satisfy $y'(x) = 0$, so $\ln(x+3) = -1$ or $x+3 = e^{-1} \simeq 0.3679$, so $x \simeq -2.6321$. When $x = e^{-1} - 3$, $y = -e^{-1}$ and is a minimum.
Second derivative $y''(x) = \frac{1}{x+3} > 0$ for $x > -3$. There is no point of inflection, and the function is concave up.



Problem 2g

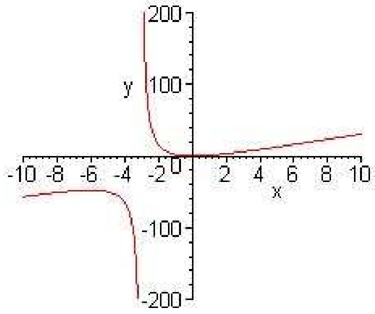


Problem 2h

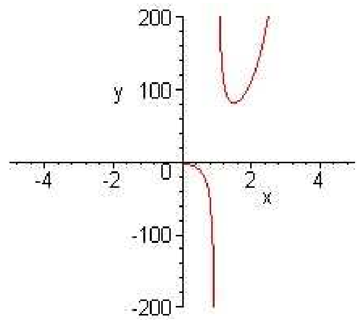
h. $y = (x-4)e^{2x}$
Domain is all x.
y-intercept: $y(0) = -4$, so $(0, -4)$.
x-intercept: Since the exponential function is not zero, $y = 0$ when $x = 4$.
Horizontal asymptote: As $x \to -\infty$, $y \to 0$, so $y = 0$ is a horizontal asymptote (looking to the left).
Derivative: By the product rule, $y'(x) = 2(x-4)e^{2x} + e^{2x} = (2x-7)e^{2x}$.
Critical points satisfy $y'(x) = 0$, so $2x - 7 = 0$ or $x = 3.5$. With $y(3.5) = -0.5e^7 \simeq -548.3$, $(3.5, -548.3)$ is a minimum.
Second derivative $y''(x) = 2(2x-7)e^{2x} + 2e^{2x} = 4(x-3)e^{2x}$.
Point of inflection ($y'' = 0$): At $x = 3$, $y(3) = -e^6 \simeq -403.4$. Thus, $(3, -402.4)$.

i. $y = \dfrac{4x^2}{x+3}$
Domain all $x \neq -3$
x and y-intercept: $(0, 0)$.
Vertical asymptote: $x = -3$
Derivative: By the quotient rule, $y'(x) = \frac{4(2x(x+3)-x^2)}{(x+3)^2} = \frac{4x(x+6)}{(x+3)^2}$.
Critical points satisfy $y'(x) = 0$, so $x = 0$ and $x = -6$. When $x = 0$, $y = 0$ and is a minimum. When $x = -6$, $y = -48$ and is a maximum.
Second derivative $y''(x) = \frac{4((x^2+6x+9)(2x+6)-(x^2+6x)(2x+6))}{(x+3)^4} = \frac{36(2x+6)}{(x+3)^4}$. There is no point of inflection, as $y''(x) = 0$ at $x = -3$, the vertical asymptote.

j. $y = \dfrac{2\,e^{2x}}{x-1}$
Domain is all $x \neq 1$.
y-intercept: $y(0) = -2$, so $(0, -2)$.
No x-intercept: The numerator is clearly never zero.
Vertical asymptote: $x = 1$.
Horizontal asymptote: As $x \to -\infty$, $y \to 0$, so $y = 0$ is a horizontal asymptote (looking to the left).
Derivative: By the product rule, $y'(x) = \frac{2((x-1)2e^{2x}-e^{2x})}{(x-1)^2} = \frac{2e^{2x}(2x-3)}{(x-1)^2}$.
Critical points satisfy $y'(x) = 0$, so $2x - 3 = 0$ or $x = 1.5$. With $y(1.5) = \frac{2e^3}{0.5} \simeq 80.342$, $(1.5, 80.342)$



Problem 2i



Problem 2j

is a minimum.
Second derivative $y''(x) = \frac{2((x^2-2x+1)(4x-4)e^{2x}-(2x-3)(2x-2)e^{2x})}{(x-1)^4} = \frac{4e^{2x}(2x^2-6x+5)}{(x-1)^3}$.
There are no points of inflection, since $y'' \neq 0$.

3. a. The temperature is given by $T(t) = 0.002t^3 - 0.09t^2 + 1.2t + 32$, which upon differentiation becomes

$$\frac{dT}{dt} = 0.006\,t^2 - 0.18\,t + 1.2.$$

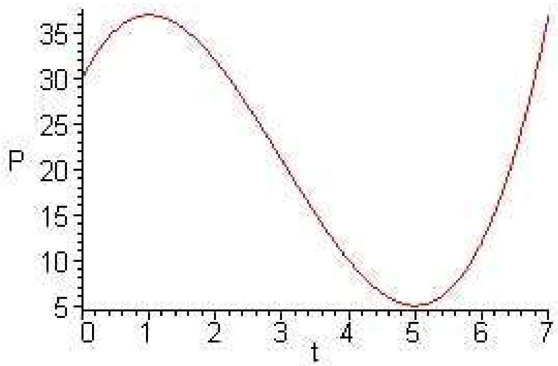
At noon, $T'(12) = 0.006(144) - 0.18(12) = -0.096$ °C/hr.

b. To find extrema, solve $T'(t) = 0.006(t^2 - 30t + 2000) = 0.006(t - 10)(t - 20) = 0$. It follows $t = 10$ and $t = 20$, so $T(10) = 2 - 9 + 12 + 32 = 37$ and $T(20) = 16 - 36 + 24 + 32 = 36$. The maximum temperature of the subject occurs at 10 AM with a temperature of 37 °C, while the minimum temperature of the subject occurs at 8 PM ($t = 20$) with a temperature of 36 °C.

4. a. $P'(t) = 3t^2 - 18t + 15$. $P'(2) = -9$ thousand algae/cc/day.

b. There is a maximum at $t = 1$ with $P(1) = 37$. There is a minimum at $t = 5$ with $P(5) = 5$. The population is increasing for $t \in (0, 1)$ and $t \in (5, 7)$. It is decreasing for $t \in (1, 5)$

c. The population at the beginning and end are $P(0) = 30$ and $P(7) = 37$. Below is the graph.



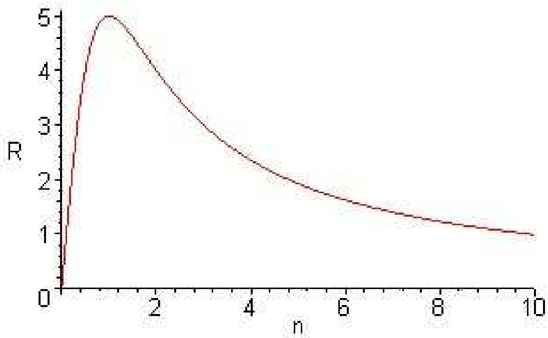
5. a. The rate of change in growth rate as a function of nutrient application is

$$\frac{dR}{dn} = -10\,\frac{n^2-1}{(1+n^2)^2}.$$

The rate of change in the growth rate when $n = 2$ is $R'(2) = -1.2$ (mm/day/mg/l)

b. There is a maximum at $n = 1$ (and a minimum occurs at $n = -1$, which is outside the domain) with $R(1) = 5$.

c. The n and R-intercept is $(0, 0)$, and there is a horizontal asymptote at $R = 0$. Below is a graph of the function.

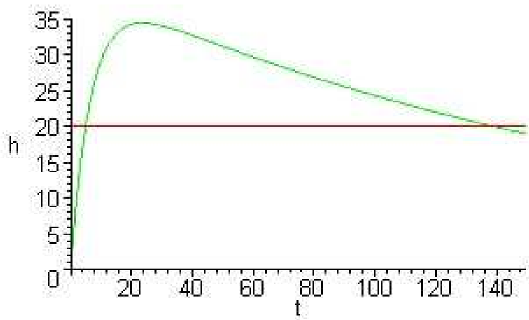


6. a. Since $h(t) = 40\left(e^{-0.005t} - e^{-0.15t}\right)$, it follows that

$$h'(t) = 40\left(-0.005e^{-0.005t} - (-0.15)e^{(} - 0.15t)\right) = 40(0.15e^{(} - 0.15t) - 0.005e^{-0.005t}).$$

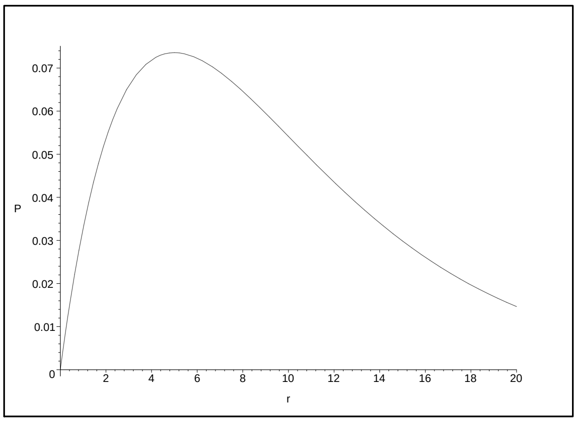
The maximum occurs when $h'(t) = 0$, so $0.15e^{(} - 0.15t) = 0.005e^{-0.005t}$ or $e^{-0.005t}e^{0.15t} = \frac{0.15}{0.005}$. Thus, $e^{(.15-.005)t} = e^{0.145t} = 30$ or $0.145t = \ln(30)$. The maximum is at $t_c = \frac{1}{0.145}\ln(30) \simeq 23.46$ days. The maximum concentration is $h(t_c) = 40\left(e^{-0.005t_c} - e^{-0.15t_c}\right) = 34.39$ ng/dl.

b. The only intercept is $(0, 0)$. There is a horizontal asymptote at $h = 0$, since $\lim_{t\to\infty} h(t) = 0$. Maple can be used to show the $h(t) = 20$ at $t = 5.0$ and 138.6, so the hormone level remains above 20 ng/dl of blood for about 134 days. The graph is shown below.



7. By the product rule, the derivative is $P'(r) = 0.04e^{-0.2r} - 0.008re^{-0.2r}$.The maximum probability occurs when the derivative is zero, $0.04e^{-0.2r} - 0.008re^{-0.2r} = 0.04e^{-0.2r}(1-0.2r)$ or $0.2r = 1$. Thus,

the maximum probability of a seed landing occurs at $r = 5$ m with a probability of $P(5) = 0.0736$. The graph of the probability density function has an intercept at $(0, 0)$ ($P(0) = 0$), a horizontal asymptote of $P = 0$ (since for large r, P becomes arbitrarily small), and a local maximum of $(5, 0.0736)$.



8. a. The equilibrium satisfies $N_e(0.8 - 0.04\ln(N_e)) = 0$. Since $N = 0$ is not in the domain. Thus, the equilibrium satisfies $0.04\ln(N_e) = 0.8$ or $\ln(N_e) = 20$. It follows that the equilibrium is $N_e = 4.852 \times 10^8$.

b. By the product rule, the derivative is $G'(N) = -N(0.04/N) + (0.8 - 0.04\ln(N)) = 0.76 - 0.04\ln(N)$. The maximum growth rate satisfies $0.76 - 0.04\ln(N) = 0$ or $\ln(N) = 19$. Thus, the maximum rate of growth occurs at $N_{max} = e^{19} = 1.785 \times 10^8$ with a maximum growth rate of $G(N_{max}) = 7.139 \times 10^6$.

9. a. The concentration of glucose is given by $g(t) = 70 + 90e^{-0.7t}$, so for it to reach 90 mg/100 ml of blood, we need $90 = 70 + 90e^{-0.7t}$ or $20 = 90e^(- 0.7t)$. It follows that $e^{0.7t} = \frac{90}{20}$ or $0.7t = \ln\left(\frac{90}{20}\right)$. this takes about $t = 2.15$ hours. Note that this function has a g-intercept of 160 and a horizontal asymptote of $g = 70$. The graph for the concentration of glucose in the blood is below.

b. The rate of change of glucose per hour is

$$\frac{dg}{dt} = 0 + 90(-0.7)e^{-0.7t} = -63e^{-0.7t}.$$

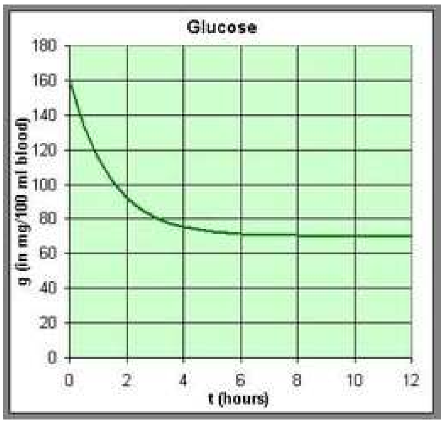
At $t = 1$, $g'(1) = -63e^{-0.7} = -31.28$ mg/100 ml of blood/hour.

c. The level of insulin satisfies the function $i(t) = 10(e^{-0.4t} - e^{-0.5t})$, so

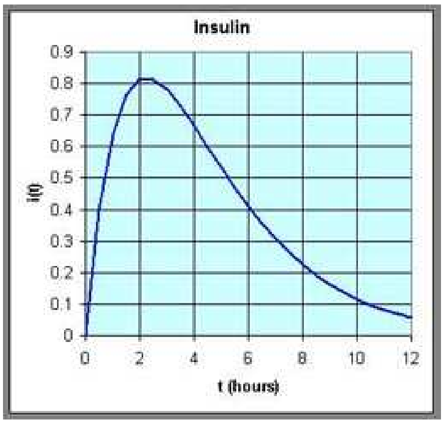
$$i'(t) = 10(-0.4e^{-0.4t} + 0.5e^{-0.5t}) = 5e^{-0.5t} - 4e^{-0.4t}.$$

The concentration is maximum where $i'(t) = 0$, so $5e^{-0.5t} = 4e^{-0.4t}$ or $\frac{5}{4} = e^{-0.4t}e^{0.5t} = e^{0.1t}$. It follows that $t = 10\ln\left(\frac{5}{4}\right) = 2.23$ hr. The maximum concentration is $i(2.23) = 10(e^{-0.4(2.23)} - e^{-0.5(2.23)}) = 0.819$. This graph starts at $(0, 0)$ and asymptotically approaches zero for large time. A graph of the insulin concentration is below also.

d. The rate of change of insulin per hour was computed above ($i'(t)$). The rate of change at $t = 1$ is $i'(1) = 5e^{-0.5} - 4e^{-0.4} = 0.351$ units/hour.



glucose



insulin

10. The radioactive decay of white lead (^{210}Pb) satisfies the equation $R(t) = R_0 e^{-kt}$. With a half-life of 22 years, we have $R_0/2 = R_0 e^{-22k}$, so $e^{22k} = 2$ or $22k = \ln(2)$. Thus, the decay constant $k = \frac{\ln(2)}{22} = 0.03151$ yr^{-1}. If the painting has 5% of the original amount of ^{210}Pb left, then $.05R_0 = R_0 e^{-0.03151t}$ so $t = \frac{\ln(0.05)}{0.03151} = 95.1$ years old.

11. a. The colony of *Escherichia coli* satisfies $P(t) = 1000e^{0.01t}$, so to find doubling time we solve $1000e^{0.01t} = 2000$ or $e^{0.01t} = 2$. Thus, the doubling time is $0.01t = \ln(2)$ or $t = 100\ln(2) = 69.3$ min.

b. The mutant satisfies $M(t) = e^{kt}$ and doubles in 25 min. It follows that $e^{25k} = 2$ or $k = \frac{\ln(2)}{25} = 0.02773$. If the mutant colony is 20% of the population of the colony, then the original population is 4 times the mutant population. (20% mutant and 80% original). Thus, we must solve $1000e^{0.01t} = 4e^{kt}$ or $e^{kt}e^{-0.01t} = e^{(0.02773-0.01)t} = e^{0.01773t} = 250$. Thus, $0.01773t = \ln(250)$ or $t = \frac{\ln(250)}{k-0.01} = 311.5$ min.

c. The original population at $t = 500$ is $P(500) = 1000e^5 = 148,413$ bacteria. The mutant population is $M(500) = e^{500k} = 1,048,576$ bacteria. The rate of growth of the original population is $\frac{dP}{dt} = 1000(0.01)e^{0.01t} = 10e^{0.01t}$, which at $t = 500$ gives $P'(500) = 10e^5 = 1,484$ bacteria/min. The rate of growth of the mutant colony is $\frac{dM}{dt} = ke^{kt} = 0.02773e^{0.02773t}$, which at $t = 500$ gives $M'(500) = ke^{500k} = 0.02772e^13.86 = 29,073$ bacteria/min.

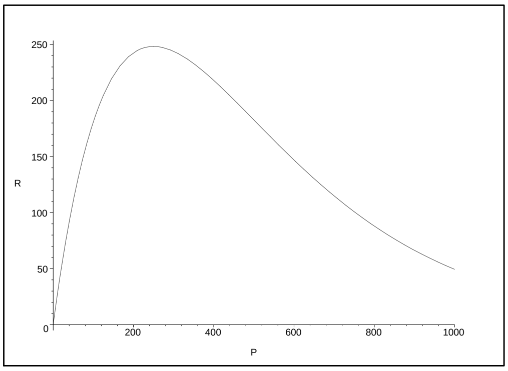
12. a. The derivative of the Ricker's updating function is

$$\frac{dR}{dP} = 2.7\,e^{-0.004P}(1 - 0.004\,P).$$

The R and P-intercept is the origin, $(0,0)$, and there is a horizontal asymptote at $R = 0$. There is a relative maximum at $P = 250$ with $R(P) = 675\,e^{-1} = 248.32$. Below is the graph.

b. The equilibria satisfy $P_e = 2.7P_e e^{-0.004P_e}$, so either $P_e = 0$ or $1 = 2.7e^{-0.004P_e}$. The latter gives $e^{0.004P_e} = 2.7$ or $P_e = 250\ln(2.7) = 248.31$.

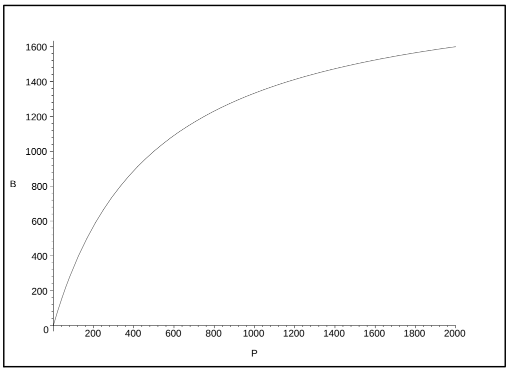
c. If $P_e = 0$, then $R'(0) = 2.7 > 1$, so the equilibrium at $P_e = 0$ is unstable. If $P_e = 248.31$, then $R'(248.31) = 0.006748 < 1$, so the equilibrium at $P_e = 248.31$ is stable.



13. a. The derivative of the Beverton-Holt's updating function is

$$\frac{dB}{dP} = \frac{4(1+0.002P) - 4P(0.002)}{(1+0.002P)^2} = \frac{4}{(1+0.002P)^2}.$$

The B and P-intercept is the origin, $(0,0)$, and there is a horizontal asymptote at $B = 2000$. There are no extrema as this function is strictly increasing to its horizontal asymptote. Below is the graph.



b. The equilibria satisfy

$$P_e = \frac{4P_e}{1+0.002P_e},$$

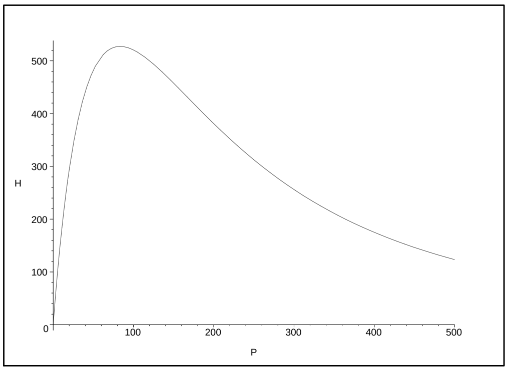
so either $P_e = 0$ or $1 + 0.002P_e = 4$. The latter gives $P_e = 1500$.

c. If $P_e = 0$, then $B'(0) = 4 > 1$, so the equilibrium at $P_e = 0$ is unstable. If $P_e = 1500$, then $B'(1500) = 0.25 < 1$, so the equilibrium at $P_e = 1500$ is stable.

14. a. The derivative of Hassell's updating function is

$$\frac{dH}{dP} = 20\frac{(1+0.004P)^4 \cdot 1 - 4P(1+0.004P)^3(0.004)}{(1+0.004P)^8} = \frac{20(1-0.012P)}{(1+0.004P)^5}.$$

The H and P-intercept is the origin, $(0,0)$, and there is a horizontal asymptote at $H = 0$. There is a critical point at $P_c = \frac{1}{0.012} = 83.333$ with $H(P_c) = 527.34$. This is clearly a maximum. Since the power of the denominator exceeds that of the numerator, there is a horizontal asymptote of $H = 0$. Below is the graph.



b. The equilibria satisfy

$$P_e = \frac{20P_e}{(1+0.004P_e)^4},$$

so either $P_e = 0$ or $(1+0.004P_e)^4 = 20$. The latter gives $P_e = 278.7$.

c. If $P_e = 0$, then $H'(0) = 20 > 1$, so the equilibrium at $P_e = 0$ is unstable. If $P_e = 278.7$, then $H'(278.7) = -1.1085 < -1$, so the equilibrium at $P_e = 278.7$ is unstable and oscillatory.

15. a. The equilibria satisfy

$$P_e = \frac{2P_e}{1+0.0025P_e^2} \quad \text{or} \quad P_e(1+0.0025P_e^2) = 2P_e.$$

Thus, either $P_e = 0$ or $1+0.0025P_e^2 = 2$. The latter implies that $0.0025P_e^2 = 1$ or $P_e^2 = 400$. Thus, $P_e = \pm 20$, but since the population density cannot be negative $P_e = 20$.

b. From the quotient rule,

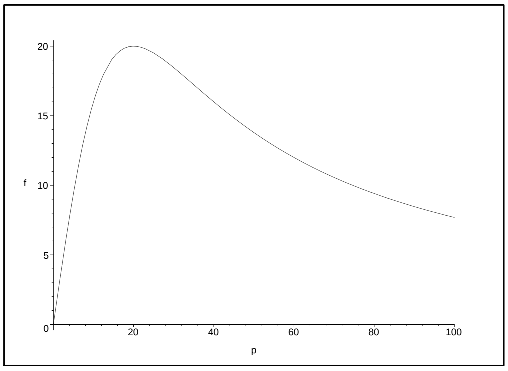
$$\begin{aligned} f'(P_n) &= \frac{(1+0.0025P_n^2)2 - 2P_n(0.005P_n)}{\left(1+0.0025\,{P_n}^2\right)^2} \\ &= \frac{2-0.005\,{P_n}^2}{\left(1+0.0025\,{P_n}^2\right)^2}. \end{aligned}$$

The maximum occurs when $f'(P_n) = 0$, which is when the numerator above is zero. Thus, $2 - 0.005P_n^2 = 0$ or $P_n^2 = 400$. It follows that the maximum mitotic increase occurs at $P_n = 20$, which is also the equilibrium.

c. A sketch of $f(P)$ is below. The only intercept is $(0,0)$. As $P_n \to \infty$, the denominator of $f(P_n)$ gets larger faster than the numerator (higher power of P_n), so $f(P_n) \to 0$, so there is a horizontal asymptote at $P_{n+1} = 0$. From Part b., the maximum occurs at $(20, 20)$.

16. a. By the quotient rule, the derivative is

$$\begin{aligned} Y'(t) &= \frac{(1+19e^{-0.1t})0 - 1000((-1.9)e^{-0.1t})}{(1+19e^{-0.1t})^2} \\ &= \frac{1900e^{-0.1t}}{(1+19e^{-0.1t})^2} = \frac{1900e^{-0.1t}}{1+38e^{-0.1t}+361e^{-0.2t}}. \end{aligned}$$



The second derivative is

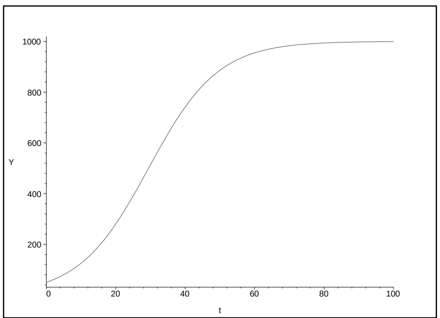
$$\begin{aligned} Y''(t) &= \frac{-190e^{-0.1t}(1+38e^{-0.1t}+361e^{-0.2t}) - 1900e^{-0.1t}(-3.8e^{-0.1t}-72.2e^{-0.2t})}{(1+19e^{-0.1t})^4} \\ &= \frac{190e^{-0.1t}(19e^{-0.1t}-1)}{(1+19e^{-0.1t})^3}. \end{aligned}$$

The second derivative is 0 when $19e^{-0.1t} - 1 = 0$ or $e^{0.1t} = 19$. $t = 10\ln(19) = 29.44$. Thus, there is a point of inflection at $(29.44, 500)$.

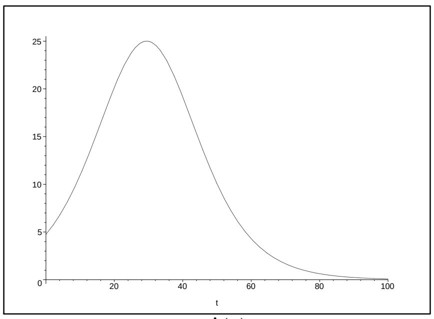
b. Only intercept is $(0, 50)$. As $t \to \infty$, $e^{-0.1t} \to 0$, so $Y(t) \to 1000$, which gives a horizontal asymptote of $Y = 1000$. A graph of $Y(t)$ is below to the left. Since the population starts at 50, it doubles when it reaches 100. Solving $Y(t) = \frac{1000}{1+19e^{-0.1t}} = 100$ gives $1 + 19e^{-0.1t} = 10$, so $e^{0.1t} = \frac{19}{9}$. Thus, this population doubles when $t = 10\ln\left(\frac{19}{9}\right) = 7.47$ hr.

c. $Y(t)$ is increasing most rapidly at the point of inflection, so $t = 29.44$ hr. Substituting this value into the derivative gives the population increasing at a rate of 25 yeast/cc/hr. The only intercept is $(0, 4.75)$. Since the numerator has a decaying exponential function, the horizontal asymptote is $Y' = 0$. A sketch of $Y'(t)$ is below to the right. The maximum for $Y'(t)$ is $(29.44, 25)$.

d. The Malthusian growth model doubles when it reaches 100. Solving $100 = 50e^{0.1t}$ gives $e^{0.1t} = 2$ or $t = 10\ln(2)$. Thus, the doubling time for the Malthusian growth model is $t = 6.93$ hr.



$Y(t)$



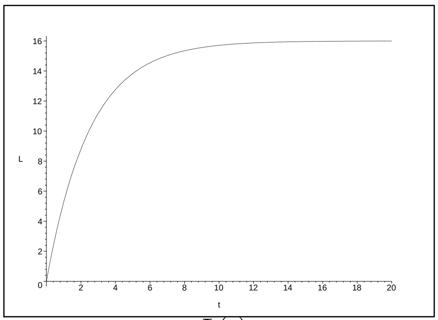
$Y'(t)$

17. a. From the von Bertalanffy equation, it is easy to see that the graph passes through the origin, giving the t and L-intercepts to both be 0. As $t \to \infty$, $L(t) \to 16$, so there is a horizontal asymptote of $L = 16$. The graph of the length of the sculpin is below to the left.

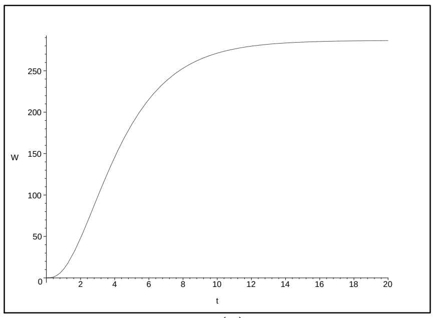
b. The composite function satisfies:

$$W(t) = 0.07\left(16(1 - e^{-0.4t})\right)^3 = 286.72(1 - e^{-0.4t})^3.$$

This function again passes through the origin, and it is easy to see that it has a horizontal asymptote at $W = 286.72$.



$L(t)$



$W(t)$

c. We apply the chain rule to differentiate $W(t)$. The result is

$$W'(t) = 3 \cdot 286.72(1 - e^{-0.4t})^2(0.4)e^{-0.4t} = 344.064(1 - e^{-0.4t})^2 e^{-0.4t}.$$

The second derivative combines the product rule and the chain rule, giving:

$$\begin{aligned} W''(t) &= 344.064\left(-0.4(1 - e^{-0.4t})^2 e^{-0.4t} + 2(1 - e^{-0.4t})0.4e^{-0.4t}e^{-0.4t}\right) \\ &= 137.6256(1 - e^{-0.4t})e^{-0.4t}\left(-(1 - e^{-0.4t}) + 2e^{-0.4t}\right) \\ &= 137.6256(1 - e^{-0.4t})e^{-0.4t}\left(3e^{-0.4t} - 1\right). \end{aligned}$$

The point of inflection is when the sculpin has its maximum weight gain, and this occurs when

$$W''(t) = 137.6256(1 - e^{-0.4t})e^{-0.4t}\left(3e^{-0.4t} - 1\right) = 0.$$

or

$$(3e^{-0.4t} - 1) = 0 \quad \text{or} \quad e^{0.4t} = 3 \quad \text{or} \quad t = \frac{5\ln(3)}{2} \simeq 2.7465.$$

The maximum weight gain is

$$W'(2.7465) = 50.97 \text{ g/yr}.$$