1. a. The 3 populations are $p_1 = 700$, $p_2 = 860$, and $p_3 = 988$.

b. The equilibrium is $p_e = 1500$. The equilibrium is stable.

2. a. The population of herbivores satisfies the model $P_{n+1} = 1.1P_n$. Since $P_0 = 100$, then $P_1 = 1.1(100) = 110$ and $P_2 = 1.1(110) = 121$. The population doubles when $2P_0 = (1.1)^n P_0$ or $(1.1)^n = 2$. Thus, $n\ln(1.1) = \ln(2)$ or $n = \frac{\ln(2)}{\ln(1.1)} \simeq 7.2725$. It takes about 7.3 years for the population to double.

b. For the discrete logistic growth model, $P_{n+1} = 1.1P_n - 0.0005P_n^2$ with $P_0 = 100$, then $P_1 = 1.1(100) - 0.0005(100)^2 = 105$ and $P_2 = 1.1(105) - 0.0005(105)^2 = 109.9875$.

c. At equilibrium, $P_e = 1.1P_e - 0.0005P_e^2$ or $0.0005P_e^2 - 0.1P_e = 0.0005P_e(P_e - 200) = 0$. From this factored form, it follows that the equilibria are $P_e = 0$ or $P_e = 200$.

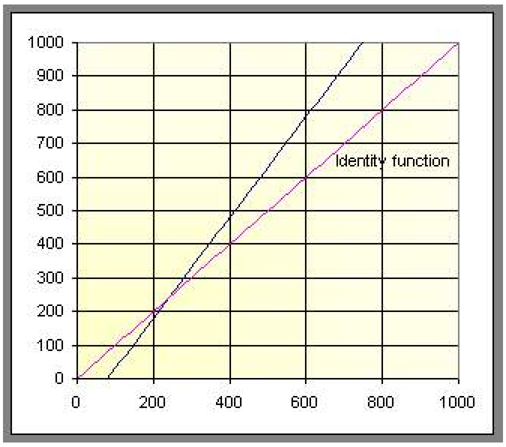
3. a. The breathing fraction is $q = 0.120536$, and the functional reserve capacity is $V_r = 2188.9$ ml.

b. The concentration of Helium in the next two breaths are $c_2 = 39.85$ and $c_3 = 35.67$. The equilibrium concentration is $c_e = \gamma = 5.2$ ppm of He, which is a stable equilibrium.

4. a. For the Malthusian growth model with dispersion, $P_{n+1} = (1+r)P_n - \mu$, $r = 0.5$ and $\mu = 120$. The populations in the next two weeks ar $P_3 = 1117.5$ and $P_4 = 1556.25$.

b. The equilibrium is $P_e = 240$, and it is unstable.

c. The graph of the updating function and identity map, $P_{n+1} = P_n$, are shown below. The only point of intersection occurs at the equilibrium found above.



5. a. From the breathing model, $c_{n+1} = (1-q)c_n + q\gamma$ and the data $c_0 = 400$, $c_1 = 352$, and $c_2 = 310$, we find the constants q and γ by substitution and the simultaneous solution of two equations and two unknowns. We have

$$352 = 400(1-q) + q\gamma \qquad \text{and} \qquad 310 = 352(1-q) + q\gamma.$$

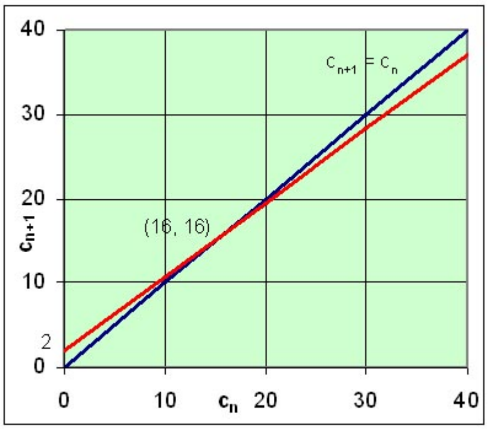
Subtracting the second equation from the first gives $42 = 48(1-q)$ or $1 - q = \frac{42}{48} = \frac{7}{8}$. Thus, $q = \frac{1}{8}$. This value is substituted into the first equation above to give $352 = 400\frac{7}{8} + \frac{1}{8}\gamma$, which gives $\gamma = 16$.

Thus, the model becomes $c_{n+1} = \frac{7}{8}c_n + 2$, and the next 2 breaths satisfy

$$\begin{aligned} c_3 &= \frac{7}{8}(310) + 2 = 273.25 \\ c_4 &= \frac{7}{8}(273.25) + 2 = 241.1 \end{aligned}$$

b. At the equilibria, $c_e = \frac{7}{8}c_e + 2$, so $\frac{1}{8}c_e = 2$ or $c_e = 16$, which is the value of γ as expected. This equilibrium is stable.

c. The graph of the updating function and identity map, $c_{n+1} = c_n$, are shown below. The only point of intersection occurs at the equilibrium, γ found above.



6. a. The next two years satisfy

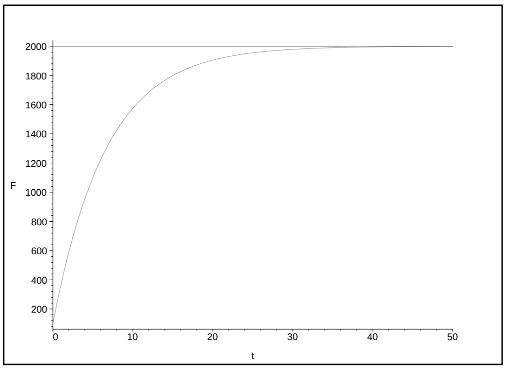
$$F_1 = 0.86(100) + 280 = 366 \qquad \text{and} \qquad F_2 = 0.86(366) + 280 = 594.8.$$

At equilibrium, $F_e = 0.86\,F_e + 280$ or $F_e = 2000$. This is a stable equilibrium. (The slope $a = 0.86 < 1$.)

b. The F-intercept is 100, and there is a horizontal asymptote at $F = 2000$. Below is the graph of this function.

c. Since $F(6) = 1227.5176$ and $F(5) = 1102.50355$, then the slope of the secant line is given by

$$\frac{F(6) - F(5)}{6 - 5} = 125.01.$$



Since $F(5.1) = 1115.8655$ and $F(5) = 1102.50355$, then the slope of the secant line is given by

$$\frac{F(5.1) - F(5)}{5.1 - 5} = 133.62.$$

d. Since $F(5.001) = 1102.63816$ and $F(5) = 1102.50355$, then the slope of the secant line is given by

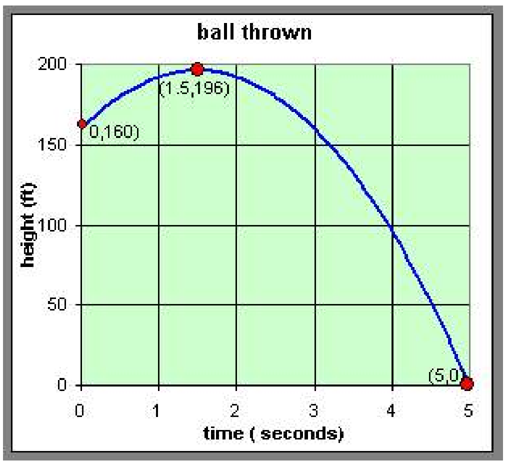
$$\frac{F(5.001) - F(5)}{5.001 - 5} = 134.6.$$

It follows that the tangent line satisfies

$$F = 134.6(t - 5) + 1102.5 = 134.6\,t + 429.4.$$

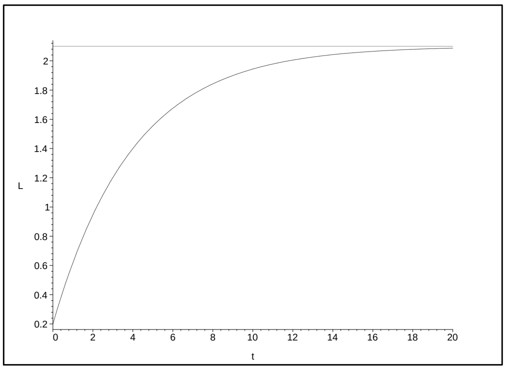
7. a. The average velocity over the for $t \in [0, 2]$ is 16 ft/sec. The average velocity over the for $t \in [1, 1.2]$ is 12.8 ft/sec. The average velocity over the for $t \in [1, 1.01]$ is 15.84 ft/sec.

b. The ball hits the ground at 5 sec with an approximate velocity of $v_{ave} = \frac{h(5)-h(4.999)}{0.001} = -111.984$ ft/sec. The graph is below.



8. a. Asymptotically, the leopard shark can reach 2.1 m. The length of the leopard shark at birth is 0.2 m, at 1 yr is 0.62 m, at 5 yr is 1.56 m, and at 10 yr is 1.94 m. The maximum length is 2.1 m. The shark reaches 90% of its maximum length at $t = 8.81$ yr. The graph is below.

b. The average growth rate for $t \in [1, 5]$ is $g_{ave} = 0.2338$ m/yr. The average growth rate for $t \in [5, 10]$ is $g_{ave} = 0.07768$ m/yr. The average growth rate for $t \in [5, 6]$ is $g_{ave} = 0.1204$ m/yr. The average growth rate for $t \in [5, 5.01]$ is $g_{ave} = 0.1359$ m/yr. This last approximation is the best approximation to the derivative (which has the value of $L'(5) = 0.1361$ m/yr).

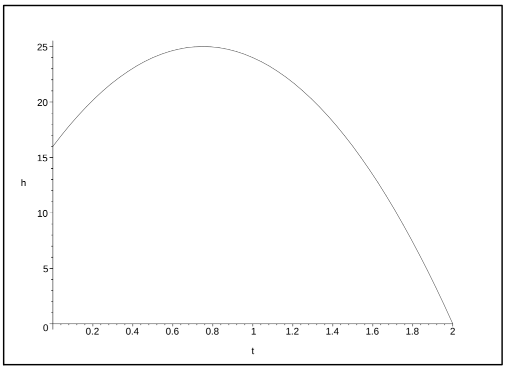


9. a. The serval can catch any bird flying at heights from 16 to 25 ft or up to 9 ft above the serval.

b. The average velocity of the serval for $t \in [0, \frac{1}{4}]$ is $v_{ave} = 20$ ft/sec. The average velocity of the serval for $t \in [\frac{1}{2}, 1]$ is $v_{ave} = 0$ ft/sec. The average velocity of the serval for $t \in [1, \frac{5}{4}]$ is $v_{ave} = -12$ ft/sec.

c. The instantaneous velocity at $t = 1$ satisfies $v(1) = -8 - 16\Delta t$. As $\Delta t \to 0$, $v(1) = -8$ ft/sec.

d. The serval hits the ground at $t = 2$. A graph of the height of the serval is below.



10. a. The vertical velocity is $v_0 = 420\sqrt{2} \simeq 593.97$ cm/sec. The impala is in the air for $t = \frac{6\sqrt{2}}{7} \simeq 1.21218$ sec.

b. The average velocity for the impala between $t = 0$ and $t = 0.5$ is $v_{ave} = 420\sqrt{2} - 245 \simeq$

348.97 cm/sec.

11. a. The slope of the secant line for $f(x) = 2x - x^2$ is $m_s = -2 - \Delta x$ through the points $(2, 0)$ and $(2 + \Delta x, f(2 + \Delta x))$.

b. The slope of the tangent line is $m_t = -2$. Thus, the value of the derivative of $f(x)$ at $x = 2$ is -2. The equation of the tangent line is

$$y = -2x + 4.$$

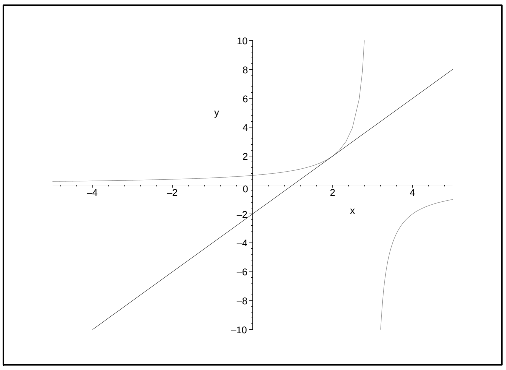
12. a. There is a vertical asymptote at $x = 3$ and a horizontal asymptote at $y = 0$. The y-intercept is $(0, \frac{2}{3})$. The graph of the function is shown below (with its tangent line).

b. The slope of the secant line for $f(x) = \dfrac{2}{3 - x}$ is

$$m_s = \frac{2}{1 - \Delta x}.$$

c. The slope of the tangent line is $m_t = 2$. Thus, the value of the derivative of $f(x)$ at $x = 2$ is 2. The equation of the tangent line is

$$y = 2x - 2.$$



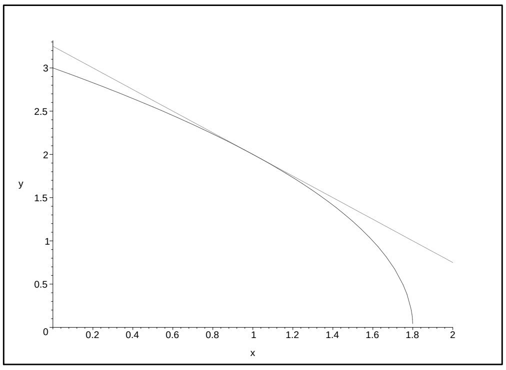
13. a. The x-intercept is $(\frac{9}{5}, 0)$, and the y-intercept is $(0, 3)$. The graph of the function is shown below (with its tangent line).

b. The slope of the secant line for $f(x) = \sqrt{9 - 5x}$ is

$$m_s = -\frac{5}{\sqrt{4 - 5\Delta x} + 2}.$$

c. The slope of the tangent line is $m_t = -\frac{5}{4}$. Thus, the value of the derivative of $f(x)$ at $x = 1$ is $-\frac{5}{4}$. The equation of the tangent line is

$$y = -\frac{5}{4}x + \frac{13}{4}.$$



14. a. The slope of the secant line is

$$m(h) = \frac{f(2+h) - f(2)}{h} = \frac{\frac{2+h-2}{2(2+h)+2} - 0}{h} = \frac{1}{6+2h}.$$

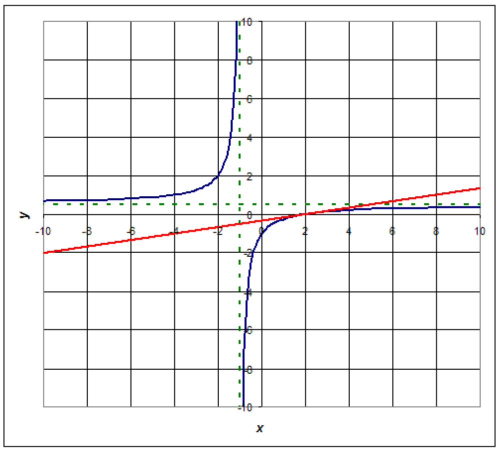
b. The slope of the tangent line

$$\lim_{h \to 0} \frac{1}{6+2h} = \frac{1}{6}.$$

The equation of the tangent line is

$$y - 0 = \frac{1}{6}(x-2) \quad \text{or} \quad y = \frac{1}{6}x - \frac{1}{3}.$$

c. The x-intercept is $x = 2$, and the y-intercept is $y = -1$. There is a vertical asymptote at $x = -1$ and a horizontal asymptote at $y = \frac{1}{2}$. Below is the graph of the function and the tangent line.



15. a. If $f(x) = \sqrt{9-3x}$, then the definition of the derivative gives

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$
\begin{aligned}
&= \lim_{h\to 0}\frac{\sqrt{9-3(x+h)}-\sqrt{9-3x}}{h}\\
&= \lim_{h\to 0}\left(\frac{\sqrt{9-3(x+h)}-\sqrt{9-3x}}{h}\right)\left(\frac{\sqrt{9-3(x+h)}+\sqrt{9-3x}}{\sqrt{9-3(x+h)}+\sqrt{9-3x}}\right)\\
&= \lim_{h\to 0}\frac{(9-3(x+h))-(9-3x)}{h(\sqrt{9-3(x+h)}+\sqrt{9-3x})}\\
&= \lim_{h\to 0}\frac{-3h}{h(\sqrt{9-3(x+h)}+\sqrt{9-3x})} = \lim_{h\to 0}\frac{-3}{\sqrt{9-3(x+h)}+\sqrt{9-3x}}\\
&= -\frac{3}{2\sqrt{9-3x}}
\end{aligned}
$$

b. If $f(x) = \frac{x}{x+2}$, then the definition of the derivative gives

$$
\begin{aligned}
f'(x) &= \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}\\
&= \lim_{h\to 0}\frac{\frac{x+h}{x+h+2}-\frac{x}{x+2}}{h}\\
&= \lim_{h\to 0}\frac{(x+2)(x+h)-(x+h+2)x}{h(x+h+2)(x+2)}\\
&= \lim_{h\to 0}\frac{(x^2+2x+hx+2h)-(x^2+hx+2x)}{h(x+h+2)(x+2)}\\
&= \lim_{h\to 0}\frac{2h}{h(x+h+2)(x+2)} = \lim_{h\to 0}\frac{2}{(x+h+2)(x+2)}\\
&= \frac{2}{(x+2)^2}
\end{aligned}
$$

16. We use the power rule of differentiation to give:

a. First rewrite

$$f(x) = 4\,x^5 - 2\,x + 4 - 2x^{-3},$$

then

$$f\,'(x) = 20\,x^4 - 2 + 6x^{-4} = 20\,x^4 - 2 + \frac{6}{x^4}.$$

b. First rewrite

$$g(t) = 12\,t^{1.2} - 5\,t^3 - 4\,t^{-1/2} - 7,$$

then

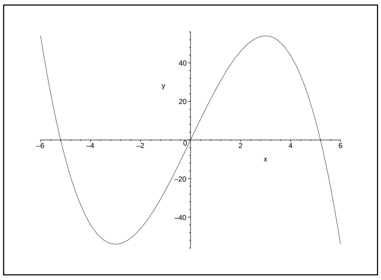
$$g\,'(t) = 14.4\,t^{0.2} - 15\,t^2 + 2\,t^{-3/2} = 14.4\,t^{0.2} - 15\,t^2 + \frac{2}{t^{3/2}}.$$

17. The domain of this function is $-4 \leq x < 0$ and $0 < x \leq 4$. The function is undefined at $x = 0$, but for the other integers, $f(-3) = 8$, $f(-2) = 1$, $f(-1) = 2$, $f(1) = 5$, $f(2) = 1$, and $f(3) = 1$. The limit fails to exist at $x = -2$ and $x = 0$.

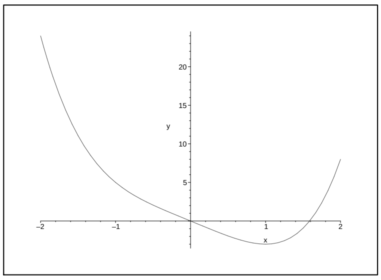
$$\begin{aligned} \lim_{x \to -3} f(x) &= 8 \\ \lim_{x \to -1} f(x) &= 2 \\ \lim_{x \to 1} f(x) &= 2 \\ \lim_{x \to 2} f(x) &= 1 \\ \lim_{x \to 3} f(x) &= 6 \end{aligned}$$

This function is continuous at all values of x with $-4 < x < 4$, except $x = -2, 0, 1$, and 3.

18. a. $y = 27x - x^3$
Domain is all x.
y-intercept: $y(0) = 0$, so $(0, 0)$.
x-intercepts: $27x - x^3 = x(27 - x^2) = 0$, so $x = 0$ and $x = \pm\sqrt{27} = \pm 3\sqrt{3}$.
No asymptotes
Derivative $y'(x) = 27 - 3x^2$
Extrema are where $y'(x) = -3(x^2 - 9) = 0$, so $x = \pm 3$. With $y(-3) = 27(-3) - (-3)^3 = -54$ and $y(3) = 54$. Thus, $(3, 54)$ is a maximum, and $(-3, -54)$ is a minimum.
Second derivative $y''(x) = -3(2)x = -6x$.
Point of inflection ($y'' = 0$): At $x = 0$ or $(0, 0)$.



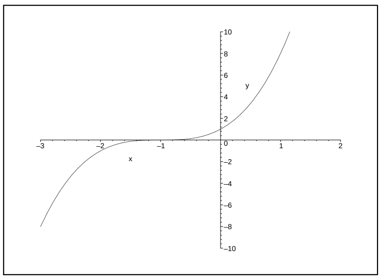
Problem 2a

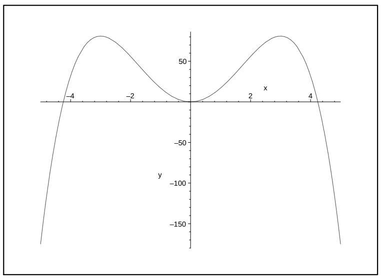


Problem 2b

b. $y = x^4 - 4x$
Domain is all x.
y-intercept: $y(0) = 0$, so $(0, 0)$.
x-intercept: $x^4 - 4x = x(x^3 - 4) = 0$, so $x = 0$ and $x = \sqrt[3]{4} \simeq 1.587$.
No asymptotes
Derivative $y'(x) = 4x^3 - 4$
Extrema are where $y'(x) = 4(x^3 - 1) = 0$, so $x = 1$. With $y(1) = 1^4 - 4(1) = -3$, $(1, -3)$ is a minimum.
Second derivative $y''(x) = 12x^2$.
Point of inflection ($y'' = 0$): At $x = 0$ or $(0, 0)$.

c. $y = x^3 + 3x^2 + 3x + 1$
Domain is all x.
y-intercept: $y(0) = 1$, so $(0, 1)$.
x-intercept: $x^3 + 3x^2 + 3x + 1 = (x + 1)^3 = 0$, so $x = -1$.
No asymptotes
Derivative $y'(x) = 3x^2 + 3(2)x + 3 = 3x^2 + 6x + 3$
Extrema are where $y'(x) = 3(x + 1)^2 = 0$, so $x = -1$ is a critical point. $y(-1) = 0$, but $(-1, 0)$ is a saddle point (neither maximum or minimum.
Second derivative $y''(x) = 3(2)x + 6 = 6x + 6$.
Point of inflection ($y'' = 0$): At $x = -1$ or $(-1, 0)$.


Problem 2c

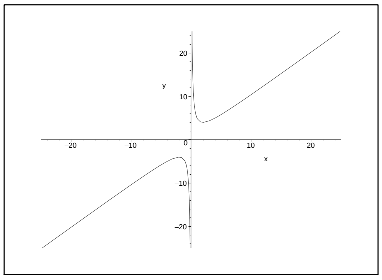

Problem 2d

d. $y = 18x^2 - x^4$
Domain is all x.
y-intercept: $y(0) = 0$, so $(0, 0)$.
x-intercept: $x^2(18 - x^2) = -x^2(x + 3\sqrt{2})(x - 3\sqrt{2}) = 0$, so $x = 0$ and $x = \pm 3\sqrt{2}$.
No asymptotes
Derivative $y'(x) = 36x - 4x^3 = 4x(9 - x^2)$
Critical points satisfy $y'(x) = -4x(x^2 - 9) = 0$, so $x = 0, \pm 3$. With $y(0) = 0$, $(0, 0)$ is a minimum.
When $x = \pm 3$,$y(\pm 3) = 81$, so there are local maxima at $(-3, 81)$ and $(3, 81)$.
Second derivative $y''(x) = 36 - 12x^2 = 12(3 - x^2)$.
Point of inflection ($y'' = 0$): At $x = \pm\sqrt{3}$, giving $(\pm\sqrt{3}, 45)$.

e. $y = x + \dfrac{4}{x} = x + 4x^{-1}$
Domain is all $x \neq 0$.
Since there is a vertical asymptote at $x = 0$, there is no y-intercept.
We solve $y = \frac{x^2+4}{x} = 0$ or $x^2 + 4 = 0$, so no x-intercepts.
Derivative $y'(x) = 1 - 4x^{-2} = \frac{x^2-4}{x^2}$
Critical points satisfy $y'(x) = 0$, so $x^2 - 4 = 0$ or $x = \pm 2$. With $y(-2) = -4$, $(-2, -4)$ is a local maximum. With $y(2) = 4$, $(2, 4)$ is a local minimum.
Second derivative $y''(x) = 8x^{-3}$, which is never zero, so no points of inflection.

19. a. The temperature is given by $T(t) = 0.002t^3 - 0.09t^2 + 1.2t + 32$, which upon differentiation becomes

$$\frac{dT}{dt} = 0.006\,t^2 - 0.18\,t + 1.2.$$



At noon, $T'(12) = 0.006(144) - 0.18(12) = -0.096$ °C/hr.

b. To find extrema, solve $T'(t) = 0.006(t^2 - 30t + 2000) = 0.006(t-10)(t-20) = 0$. It follows $t = 10$ and $t = 20$, so $T(10) = 2 - 9 + 12 + 32 = 37$ and $T(20) = 16 - 36 + 24 + 32 = 36$. The maximum temperature of the subject occurs at 10 AM with a temperature of 37 °C, while the minimum temperature of the subject occurs at 8 PM ($t = 20$) with a temperature of 36 °C.

20. a. $P'(t) = 3t^2 - 18t + 15$. $P'(2) = -9$ thousand algae/cc/day.

b. There is a maximum at $t = 1$ with $P(1) = 37$. There is a minimum at $t = 5$ with $P(5) = 5$. The population is increasing for $t \in (0,1)$ and $t \in (5,7)$. It is decreasing for $t \in (1,5)$

c. The population at the beginning and end are $P(0) = 30$ and $P(7) = 37$. Below is the graph.