1. a. $H_1 = 2040$ and $H_2 = 2080.8$. In the general solution is given by $H_n = (1.02)^n H_0 = 2000(1.02)^n$.

b. The general solution is given by $G_n = (1.03)^n G_0 = 200(1.03)^n$. It takes 23.5 generations for the population to double.

c. The populations are equal in 236 generations.

2. a. The annual growth rate is $r = 0.011753$ (about 1.2% per year) and the general equation is $P_n = (1.011753)^n 179.3$.

b. The model predicts a population of 286.1 million in the year 2000. The error between this and the actual population is 1.7%.

c. The population will double about 59.3 years after 1960, or about the year 2019.

3. a. For France, the growth rate is $r = 0.051948$ (about 5.2% per decade) and the general equation is $P_n = (1.051948)^n 53.9$.

b. The population predictions are 59.6 million in the year 2000 and 66 million in the year 2020. The error in the year 2000 is 0.34%.

c. In Kenya, the growth rate is $r = 0.4491$ (about 44.9% per decade) and the general equation is $P_n = (1.4491)^n 16.7$. The population predictions are 35.1 million in the year 2000 and 73.6 million in the year 2020. It takes 1.87 decades, or about 18.7 years, for the population of Kenya to double.

d. The populations become equal in 3.66 decades, so the population of Kenya will first exceed that of France in 2017. Population for France in 2017 is 65.0 million, while the population of Kenya in 2017 is 65.9 million.

e. It follows that the annual growth rate in France is $r = 0.00508$, while in Kenya it is $r = 0.0378$.

4. a. The 3 populations are $p_1 = 700$, $p_2 = 860$, and $p_3 = 988$.

b. The equilibrium is $p_e = 1500$. The equilibrium is stable.

5. a. The population of herbivores satisfies the model $P_{n+1} = 1.1P_n$. Since $P_0 = 100$, then $P_1 = 1.1(100) = 110$ and $P_2 = 1.1(110) = 121$. The population doubles when $2P_0 = (1.1)^n P_0$ or $(1.1)^n = 2$. Thus, $n \ln(1.1) = \ln(2)$ or $n = \frac{\ln(2)}{\ln(1.1)} \simeq 7.2725$. It takes about 7.3 years for the population to double.

b. For the discrete logistic growth model, $P_{n+1} = 1.1P_n - 0.0005P_n^2$ with $P_0 = 100$, then

$P_1 = 1.1(100) - 0.0005(100)^2 = 105$ and $P_2 = 1.1(105) - 0.0005(105)^2 = 109.9875$.

c. At equilibrium, $P_e = 1.1P_e - 0.0005P_e^2$ or $0.0005P_e^2 - 0.1P_e = 0.0005P_e(P_e - 200) = 0$. From this factored form, it follows that the equilibria are $P_e = 0$ or $P_e = 200$.

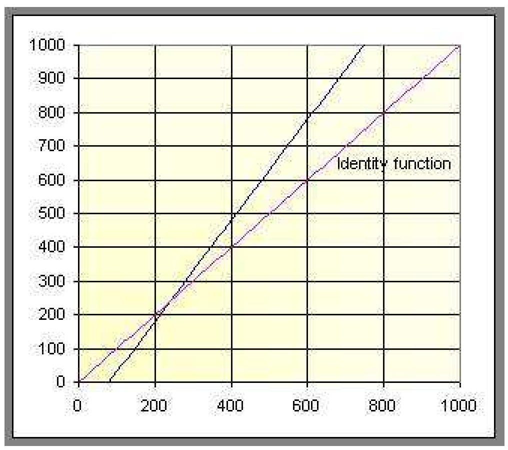
6. a. The breathing fraction is $q = 0.120536$, and the functional reserve capacity is $V_r = 2188.9$ ml.

b. The concentration of Helium in the next two breaths are $c_2 = 39.85$ and $c_3 = 35.67$. The equilibrium concentration is $c_e = \gamma = 5.2$ ppm of He, which is a stable equilibrium.

7. a. For the Malthusian growth model with dispersion, $P_{n+1} = (1+r)P_n - \mu$, $r = 0.5$ and $\mu = 120$. The populations in the next two weeks ar $P_3 = 1117.5$ and $P_4 = 1556.25$.

b. The equilibrium is $P_e = 240$, and it is unstable.

c. The graph of the updating function and identity map, $P_{n+1} = P_n$, are shown below. The only point of intersection occurs at the equilibrium found above.



8. a. From the breathing model, $c_{n+1} = (1-q)c_n + q\gamma$ and the data $c_0 = 400$, $c_1 = 352$, and $c_2 = 310$, we find the constants q and γ by substitution and the simultaneous solution of two equations and two unknowns. We have

$$352 = 400(1-q) + q\gamma \qquad \text{and} \qquad 310 = 352(1-q) + q\gamma.$$

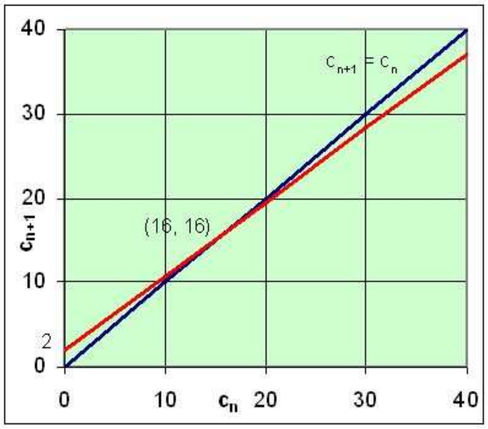
Subtracting the second equation from the first gives $42 = 48(1-q)$ or $1 - q = \frac{42}{48} = \frac{7}{8}$. Thus, $q = \frac{1}{8}$. This value is substituted into the first equation above to give $352 = 400\frac{7}{8} + \frac{1}{8}\gamma$, which gives $\gamma = 16$.

Thus, the model becomes $c_{n+1} = \frac{7}{8}c_n + 2$, and the next 2 breaths satisfy

$$\begin{aligned} c_3 &= \frac{7}{8}(310) + 2 = 273.25 \\ c_4 &= \frac{7}{8}(273.25) + 2 = 241.1 \end{aligned}$$

b. At the equilibria, $c_e = \frac{7}{8}c_e + 2$, so $\frac{1}{8}c_e = 2$ or $c_e = 16$, which is the value of γ as expected. This equilibrium is stable.

c. The graph of the updating function and identity map, $c_{n+1} = c_n$, are shown below. The only point of intersection occurs at the equilibrium, γ found above.

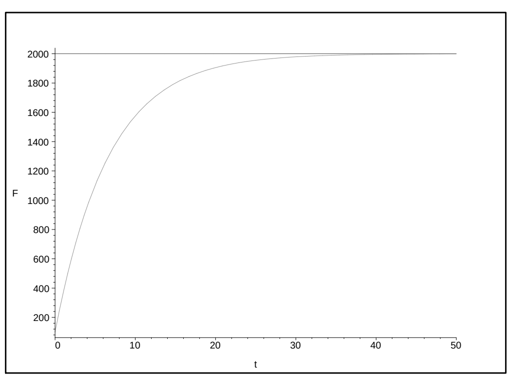


9. a. The next two years satisfy

$$F_1 = 0.86(100) + 280 = 366 \qquad \text{and} \qquad F_2 = 0.86(366) + 280 = 594.8.$$

At equilibrium, $F_e = 0.86\,F_e + 280$ or $F_e = 2000$. This is a stable equilibrium. (The slope $a = 0.86 < 1$.)

b. The F-intercept is 100, and there is a horizontal asymptote at $F = 2000$. Below is the graph of this function.



c. Since $F(6) = 1227.5176$ and $F(5) = 1102.50355$, then the slope of the secant line is given by

$$\frac{F(6) - F(5)}{6 - 5} = 125.01.$$

Since $F(5.1) = 1115.8655$ and $F(5) = 1102.50355$, then the slope of the secant line is given by

$$\frac{F(5.1) - F(5)}{5.1 - 5} = 133.62.$$

d. Since $F(5.001) = 1102.63816$ and $F(5) = 1102.50355$, then the slope of the secant line is given by

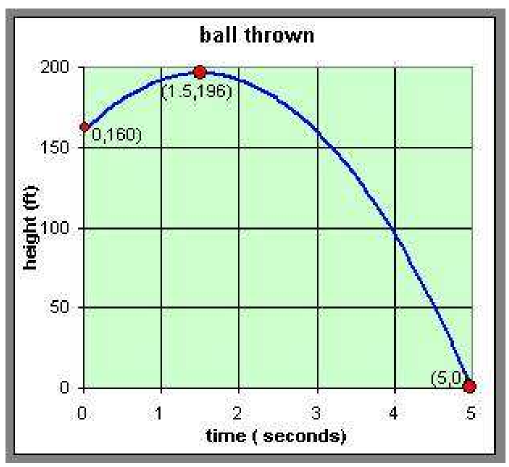
$$\frac{F(5.001) - F(5)}{5.001 - 5} = 134.6.$$

It follows that the tangent line satisfies

$$F = 134.6(t - 5) + 1102.5 = 134.6\,t + 429.4.$$

10. a. The average velocity over the for $t \in [0, 2]$ is 16 ft/sec. The average velocity over the for $t \in [1, 1.2]$ is 12.8 ft/sec. The average velocity over the for $t \in [1, 1.01]$ is 15.84 ft/sec.

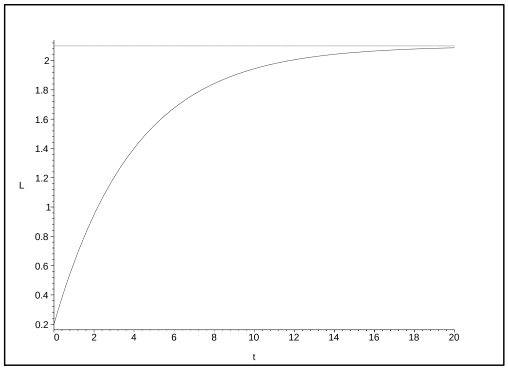
b. The ball hits the ground at 5 sec with an approximate velocity of $v_{ave} = \frac{h(5)-h(4.999)}{0.001} = -111.984$ ft/sec. The graph is below.



11. a. Asymptotically, the leopard shark can reach 2.1 m. The length of the leopard shark at birth is 0.2 m, at 1 yr is 0.62 m, at 5 yr is 1.56 m, and at 10 yr is 1.94 m. The maximum length is 2.1 m. The shark reaches 90% of its maximum length at $t = 8.81$ yr. The graph is below.

b. The average growth rate for $t \in [1, 5]$ is $g_{ave} = 0.2338$ m/yr. The average growth rate for $t \in [5, 10]$ is $g_{ave} = 0.07768$ m/yr. The average growth rate for $t \in [5, 6]$ is $g_{ave} = 0.1204$ m/yr. The average growth rate for $t \in [5, 5.01]$ is $g_{ave} = 0.1359$ m/yr. This last approximation is the best approximation to the derivative (which has the value of $L'(5) = 0.1361$ m/yr).

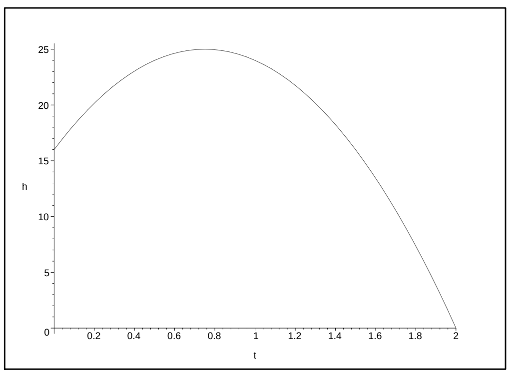
12. a. The serval can catch any bird flying at heights from 16 to 25 ft or up to 9 ft above the serval.



b. The average velocity of the serval for $t \in [0, \frac{1}{4}]$ is $v_{ave} = 20$ ft/sec. The average velocity of the serval for $t \in [\frac{1}{2}, 1]$ is $v_{ave} = 0$ ft/sec. The average velocity of the serval for $t \in [1, \frac{5}{4}]$ is $v_{ave} = -12$ ft/sec.

c. The instantaneous velocity at $t = 1$ satisfies $v(1) = -8 - 16\Delta t$. As $\Delta t \to 0$, $v(1) = -8$ ft/sec.

d. The serval hits the ground at $t = 2$. A graph of the height of the serval is below.



13. a. The vertical velocity is $v_0 = 420\sqrt{2} \simeq 593.97$ cm/sec. The impala is in the air for $t = \frac{6\sqrt{2}}{7} \simeq 1.21218$ sec.

b. The average velocity for the impala between $t = 0$ and $t = 0.5$ is $v_{ave} = 420\sqrt{2} - 245 \simeq 348.97$ cm/sec.

14. a. The slope of the secant line for $f(x) = 2x - x^2$ is $m_s = -2 - \Delta x$ through the points $(2, 0)$ and $(2 + \Delta x, f(2 + \Delta x))$.

b. The slope of the tangent line is $m_t = -2$. Thus, the value of the derivative of $f(x)$ at $x = 2$ is -2. The equation of the tangent line is

$$y = -2x + 4.$$

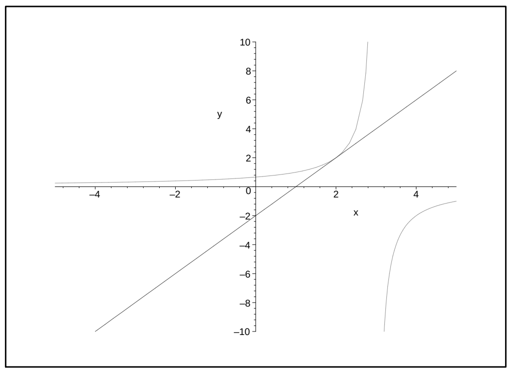
15. a. There is a vertical asymptote at $x = 3$ and a horizontal asymptote at $y = 0$. The y-intercept is $(0, \frac{2}{3})$. The graph of the function is shown below (with its tangent line).

b. The slope of the secant line for $f(x) = \dfrac{2}{3 - x}$ is

$$m_s = \frac{2}{1 - \Delta x}.$$

c. The slope of the tangent line is $m_t = 2$. Thus, the value of the derivative of $f(x)$ at $x = 2$ is 2. The equation of the tangent line is

$$y = 2x - 2.$$



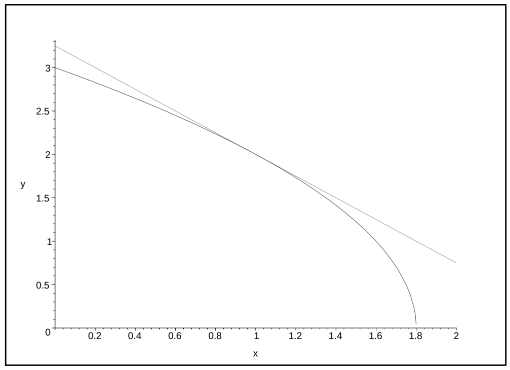
16. a. The x-intercept is $(\frac{9}{5}, 0)$, and the y-intercept is $(0, 3)$. The graph of the function is shown below (with its tangent line).

b. The slope of the secant line for $f(x) = \sqrt{9 - 5x}$ is

$$m_s = -\frac{5}{\sqrt{4 - 5\Delta x} + 2}.$$

c. The slope of the tangent line is $m_t = -\frac{5}{4}$. Thus, the value of the derivative of $f(x)$ at $x = 1$ is $-\frac{5}{4}$. The equation of the tangent line is

$$y = -\frac{5}{4}x + \frac{13}{4}.$$



17. a. If $f(x) = \sqrt{9-3x}$, then the definition of the derivative gives

$$\begin{aligned} f'(x) &= \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} \\ &= \lim_{h\to 0} \frac{\sqrt{9-3(x+h)}-\sqrt{9-3x}}{h} \\ &= \lim_{h\to 0} \left(\frac{\sqrt{9-3(x+h)}-\sqrt{9-3x}}{h}\right)\left(\frac{\sqrt{9-3(x+h)}+\sqrt{9-3x}}{\sqrt{9-3(x+h)}+\sqrt{9-3x}}\right) \\ &= \lim_{h\to 0} \frac{(9-3(x+h))-(9-3x)}{h(\sqrt{9-3(x+h)}+\sqrt{9-3x})} \\ &= \lim_{h\to 0} \frac{-3h}{h(\sqrt{9-3(x+h)}+\sqrt{9-3x})} = \lim_{h\to 0} \frac{-3}{\sqrt{9-3(x+h)}+\sqrt{9-3x}} \\ &= -\frac{3}{2\sqrt{9-3x}} \end{aligned}$$

b. If $f(x) = \frac{x}{x+2}$, then the definition of the derivative gives

$$\begin{aligned} f'(x) &= \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} \\ &= \lim_{h\to 0} \frac{\frac{x+h}{x+h+2}-\frac{x}{x+2}}{h} \\ &= \lim_{h\to 0} \frac{(x+2)(x+h)-(x+h+2)x}{h(x+h+2)(x+2)} \\ &= \lim_{h\to 0} \frac{(x^2+2x+hx+2h)-(x^2+hx+2x)}{h(x+h+2)(x+2)} \\ &= \lim_{h\to 0} \frac{2h}{h(x+h+2)(x+2)} = \lim_{h\to 0} \frac{2}{(x+h+2)(x+2)} \\ &= \frac{2}{(x+2)^2} \end{aligned}$$

18. We use the power rule of differentiation to give:

a. First rewrite

$$f(x) = 4\,x^5 - 2\,x + 4 - 2x^{-3},$$

then

$$f\,'(x) = 20\,x^4 - 2 + 6x^{-4} = 20\,x^4 - 2 + \frac{6}{x^4}.$$

b. First rewrite

$$g(t) = 12\,t^{1.2} - 5\,t^3 - 4\,t^{-1/2} - 7,$$

then

$$g\,'(t) = 14.4\,t^{0.2} - 15\,t^2 + 2\,t^{-3/2} = 14.4\,t^{0.2} - 15\,t^2 + \frac{2}{t^{3/2}}.$$

19. The domain of this function is $-4 \le x < 0$ and $0 < x \le 4$. The function is undefined at $x = 0$, but for the other integers, $f(-3) = 8$, $f(-2) = 1$, $f(-1) = 2$, $f(1) = 5$, $f(2) = 1$, and $f(3) = 1$. The limit fails to exist at $x = -2$ and $x = 0$.

$$\begin{aligned}
\lim_{x\to -3} f(x) &= 8\\
\lim_{x\to -1} f(x) &= 2\\
\lim_{x\to 1} f(x) &= 2\\
\lim_{x\to 2} f(x) &= 1\\
\lim_{x\to 3} f(x) &= 6
\end{aligned}$$

This function is continuous at all values of x with $-4 < x < 4$, except $x = -2, 0, 1$, and 3.