Give all answers to at least **4 significant figures**.

1. Over the summer a group of ecology graduate students collects data from a number of sample plots on insect populations infecting a particular crop. Below is a table showing the average data that they collected in different weeks.

Week	Population ($/m^2$)
0	57.3
1	54.1
2	61.3
4	112.7
5	143.3
7	152.6
9	119.4
10	98.6
11	78.3
13	82.5
14	85.8
15	103.6

a. Fit the best fourth order polynomial through the data of the form

$$P(t) = a_4t^4 + a_3t^3 + a_2t^2 + a_1t + a_0.$$

Be sure that all of your coefficients have 4 significant figures. Write your best fitting model to the data.

b. Differentiate $P(t)$ and give the formula for the derivative.

c. Find all relative minima and maxima during the time of the survey, $t \in [0, 15]$. Be sure to give both the values of t and P (to 4 significant figures). Also, find the absolute minimum and maximum by comparing your relative extrema to the values at the endpoints. List the absolute minimum and maximum populations and when the model predicts that they will occur.

d. The best time to treat this pest against spreading is when the population is growing most rapidly. Use the second derivative to find when the rate of growth ($\frac{dP}{dt}$) is at a maximum and at a minimum for $t \in [0, 15]$. Also, check the values of the derivative at the endpoints, then list the absolute maximum and minimum rate of growth $t \in [0, 15]$. Be sure to give the appropriate units for this growth rate.

2. An experimental drug is being tested to see the response of the immune system in a drug trial for cancer chemotherapy. Below is a table of the readings of the drug in the blood as time passes after an injection of the drug at $t = 0$ days.

Day	Drug (μg/dl)
0	8
1	7.5
2	6.2
4	5.3
6	3.7
10	2.2
15	1.8
20	0.9

a. The drug is metabolized in the liver and eliminated through the urine. It is assumed that the decay of this drug is exponential, so fits the model

$$D(t) = A\, e^{-kt}.$$

Use Excel's Trendline with an Exponential fit to the data. Give the best values of A and k that you find (to 4 significant figures). Also, give the sum of square errors between the data and the model.

b. The drug stimulates a cytokine response in the blood. Over the next 20 days the blood is measured to determine the response of the body to this new experimental drug. Below are the data for the level of cytokine in the blood.

Day	Cytokine (ng/dl)
0	0
1	8.5
2	15.9
4	25.4
6	33.1
10	38.2
15	35.7
20	30.6

The researchers use the standard model in pharmakinetics of exponential release and decay of the cytokine in the body. This model is given by

$$C(t) = B(e^{-qt} - e^{-kt}),$$

where the decay k matches the drug decay from Part a. Use Excel's solver to find the least squares best fit to the parameters B and q. As an initial guess use $B = 200$ and $q = 0.05$. Write the model with the best parameters, then state the sum of square errors between this model and the data. Give the percent error at $t = 10$ and $t = 20$ days. Find the derivative of this model ($C'(t)$). Use the techniques from class to find the time that this model predicts a maximum concentration of the cytokine and when this occurs. (Give your answers to 4 significant figures.)

c. Another researcher suggests that since the cytokines are being released by the white blood cells that a population model might be more appropriate. She suggests that the data might be better fit by a Ricker's model of the form

$$R(t) = Kt\,e^{-rt}.$$

Use Excel's solver to find the least squares best fit to the parameters K and r. As an initial guess use $K = 10$ and $r = 0.1$. Write the model with the best parameters, then state the sum of square errors between this model and the data. Which model gives the smaller sum of square errors. Give the percent error at $t = 10$ and $t = 20$ days. Find the derivative of this model ($R'(t)$). Use the techniques from class to find the time that this model predicts a maximum concentration of the cytokine and when this occurs. (Give your answers to 4 significant figures.)