1. a. Consider a model with immigration given by

$$p_{n+1} = 0.8p_n + 300,$$

with an initial population of $p_0 = 500$. Determine the populations at the next three time intervals, p_1, p_2, and p_3.

b. Find all equilibria and determine the stability of these equilibria.

2. a. A population of herbivores reproduces annually and satisfies the discrete dynamical system:

$$P_{n+1} = P_n + rP_n,$$

where $r = 0.1$ is the net growth rate. If the initial population is $P_0 = 100$ individuals, then determine how many herbivores there are in each of the next two years. How long does it take for this population to double?

b. Often there are crowding effects due to limited resources. This is often modeled by the discrete logistic growth model. Assume these herbivores satisfy the model:

$$P_{n+1} = 1.1P_n - 0.0005P_n^2.$$

If the initial population is $P_0 = 100$, then find P_1 and P_2.

c. Find both equilibria for the discrete logistic equation in Part b.

3. A man with a chronic lung problem has a tidal volume, V_i, of 300 ml. For this experiment, Helium, He, is used to determine the functional reserve capacity, V_r. (Note that $V_r = (1-q)V_i/q$.) The mathematical model gives

$$c_{n+1} = (1-q)c_n + q\gamma,$$

where $\gamma = 5.2$ ppm.

a. The man is given an enriched mixture of air to breathe that contains 50 ppm of He. Experimentally, the concentration of He in the first two measured breaths after breathing the enriched mixture are given by $c_0 = 50$ and $c_1 = 44.6$ ppm. Use c_0 and c_1 to find q, then determine the functional reserve capacity, V_r.

b. Use your model to find the expected concentration of Helium in this patient's 3^{rd} breath, c_3. What is the equilibrium concentration of Helium in the patient's lungs? What is the stability of this equilibrium concentration?

4. Below are data on the population of insect pests living in a survey area. The insect reproduces according to a Malthusian growth model and disperses (emigrates) to surrounding regions at a constant rate. The population model for this insect pest is given by

$$P_{n+1} = (1+r)P_n - \mu,$$

where r is the rate of growth (per week) and μ is the number of pests dispersing each week to surrounding regions.

a. From the data below determine the updating function for this population, *i.e.*, find r and μ. Then use this updating function to find the population of the insect pests for weeks 3 and 4.

b. Find all equilibria for this model. Based on your iterations in Part a, what is the stability of the equilibria? (If a solution moves closer to an equilibrium point, then it is probably stable. If it moves away, then it is most likely unstable.)

c. Graph the updating function along with the identity map, $P_{n+1} = P_n$. Determine all points of intersection.

Week	Insects
0	500
1	630
2	825

5. A woman with a chronic lung problem breathes a supply of air enriched with Helium. Experimental measurements show the following concentrations of the exhaled air after she resumes normal breathing in the room.

Breath Number	0	1	2
Conc. of He (ppm)	400	352	310

The concentration of Helium in the room, γ, is not known, but assumed to be constant.

a. Assume a breathing model of the form:

$$c_{n+1} = (1 - q)c_n + q\gamma.$$

Use the data above to find the constants q, the fraction of air exchanged, and γ, the ambient concentration of Helium. Then determine the concentration of Helium in the next two breaths, c_3 and c_4.

b. Find the equilibrium concentration of Helium in the subject's lungs based on this breathing model. What is the stability of this equilibrium concentration? (If a solution moves closer to an equilibrium point, then it is probably stable. If it moves away, then it is most likely unstable.)

c. Graph the updating function along with the identity map, $c_{n+1} = c_n$, showing all intercepts ($c_n \geq 0$) and points of intersection.

6. a. An ecological survey for a particular invasive species of fish is conducted at a lake. This species reproduces in the lake and is predated upon by other fish in the lake. There is also a constant immigration of the fish from a river feeding the lake. The survey finds that the population of this fish satisfies the discrete linear growth model

$$F_{n+1} = 0.86\,F_n + 280, \qquad F_0 = 100,$$

where F_n is the population of fish in the n^{th} year and $F_0 = 100$ is the initial population surveyed. Find the populations F_1 and F_2. Find the equilibrium population, F_e. Is this a stable or unstable population?

b. The discrete model above is approximated very well by the continuous model

$$F(t) = 2000 - 1900\, e^{-0.15t}.$$

Find the F-intercept and any horizonal asymptotes for $F(t)$, then sketch a graph of the function.

c. Use $F(t)$ to find the slope of the secant line between $t = 5$ and $t = 6$. Also, find the slope of the secant line between $t = 5$ and $t = 5.1$.

d. A good approximation to the slope of the tangent line (which approximates the derivative) is found from the slope of the secant line between $t = 5$ and $t = 5.001$. Find this slope and write the equation for the tangent line. (**Warning:** Keep 8-10 significant figures on your calculator.)

7. A ball, which is thrown vertically upward with a velocity of 48 ft/sec and falls under the influence of gravity without air resistance from a 160 ft cliff, satisfies the equation

$$h(t) = 160 + 48t - 16t^2,$$

where h is in feet and t is in seconds.

a. Find the average velocity of the ball between the times $t = 0$ and $t = 2$. Also, find the average velocity between the times $t = 1$ and $t = 1.2$ and between the times $t = 1$ and $t = 1.01$

b. Sketch a graph of $h(t)$, showing crucial points, including the h-intercept, the maximum height, and when the ball hits the ground. Approximate the velocity with which the ball hits the ground by finding the average velocity of the ball between the time the ball hits the ground and 0.001 sec before it hits the ground.

8. Fish continue to grow throughout their life, but their growth slows with age. The leopard shark (*Triakis semifiate*) is a common ovoviviparous, benthic shark in the San Diego waters. A good approximation to the growth of the leopard shark uses the von Bertalanffy equation for fish growth and is given by

$$L(t) = 2.1 - 1.9e^{-0.25t},$$

where L is in meters and t is in years.

a. Determine any asymptotes. Find the approximate length of a leopard shark at birth and ages 1, 5, and 10 years. Sketch a graph of the length of a leopard shark. What is the maximum length of a leopard shark and at what age does it reach 90% of that length?

b. Find the average rate of growth between the ages of 1 and 5 years, between the ages of 5 and 10 years, between the ages of 5 and 6 years, and between the ages of 5 and 5.01 years. Which of these gives the best approximation to the derivative (instantaneous rate of growth) at $t = 5$ years?

9. a. A serval is a 20-40 pound wild cat that has an incredible ability to leap and catch birds. Its predation success rate ranks among the highest of any animals. Suppose that a serval is sitting on a ledge 16 ft above the ground and can jump with a vertical velocity of 24 ft/sec. If we ignore air resistance and use an acceleration from gravity of -32 ft/sec^2, then the height of the serval above the ground, $h(t)$, is given by the formula

$$h(t) = 16 + 24t - 16t^2.$$

Find the range of heights for a bird flying above the serval that allows this predator a chance to catch it.

b. Find the average velocity of the serval for the intervals $t \in [0, \frac{1}{4}]$, $t \in [\frac{1}{2}, 1]$, and $t \in [1, \frac{5}{4}]$.

c. Find the instantaneous velocity at $t = 1$ by computing

$$v(1) = \frac{h(1 + \Delta t) - h(1)}{\Delta t},$$

and finding the limiting value for $\Delta t \to 0$.

d. Determine the time when the serval hits the ground. Sketch a graph of the height of the serval as a function of t.

10. An impala is migrating across a field that has been fenced with a 180 cm fence. To escape it needs to jump this fence. Assume that the impala jumps the fence with just enough vertical velocity, v_0 to clear it. If the height (in cm) of the impala is given by

$$h(t) = v_0 t - 490t^2.$$

a. Use the height of the fence (maximum height) and the function describing the height of the impala, $h(t)$, to determine the vertical velocity, v_0, then determine how long the impala is in the air.

b. With this value of v_0, find the average velocity of the impala between $t = 0$ and $t = 0.5$.

11. a. Find the slope of the secant line through the points $(2, f(2))$ and $(2 + \Delta x, f(2 + \Delta x))$ for the function

$$f(x) = 2x - x^2.$$

b. Let Δx get small and determine the slope of the tangent line through $(2, f(2))$, which gives the value of the derivative of $f(x)$ at $x = 2$. Give the equation of the tangent line.

12. Consider the function

$$f(x) = \frac{2}{3 - x}.$$

a. Sketch a graph of the function, including any asymptotes.

b. Find the slope of the secant line through the points $(2, f(2))$ and $(2 + \Delta x, f(2 + \Delta x))$ for the function

c. Let Δx get small and determine the slope of the tangent line through $(2, f(2))$, which gives the value of the derivative of $f(x)$ at $x = 2$. Give the equation of this tangent line.

13. Consider the function

$$f(x) = \sqrt{9 - 5x}.$$

a. Sketch a graph of the function.

b. Compute the slope of the secant line through the points $x = 1$ and $x = 1 + \Delta x$.

c. From your answer in Part b., take the limit as $\Delta x \to 0$ to obtain the slope of the tangent line, then write the equation of the tangent line at $x = 1$.

14. a. Find the slope of the secant line through the points $(2, f(2))$ and $(2+h, f(2+h))$ for the function

$$f(x) = \frac{x-2}{2x+2}.$$

b. Continuing from Part a, let h get small and determine the slope of the tangent line through $(2, f(2))$, which gives the value of the derivative of $f(x)$ at $x = 2$. Give the equation of this tangent line.

c. Sketch a graph of $f(x)$ giving x and y-intercepts, vertical and horizontal asymptotes, and including the tangent line at $(2, f(2))$.

15. Use the definition of the derivative to find the derivative of the following functions.

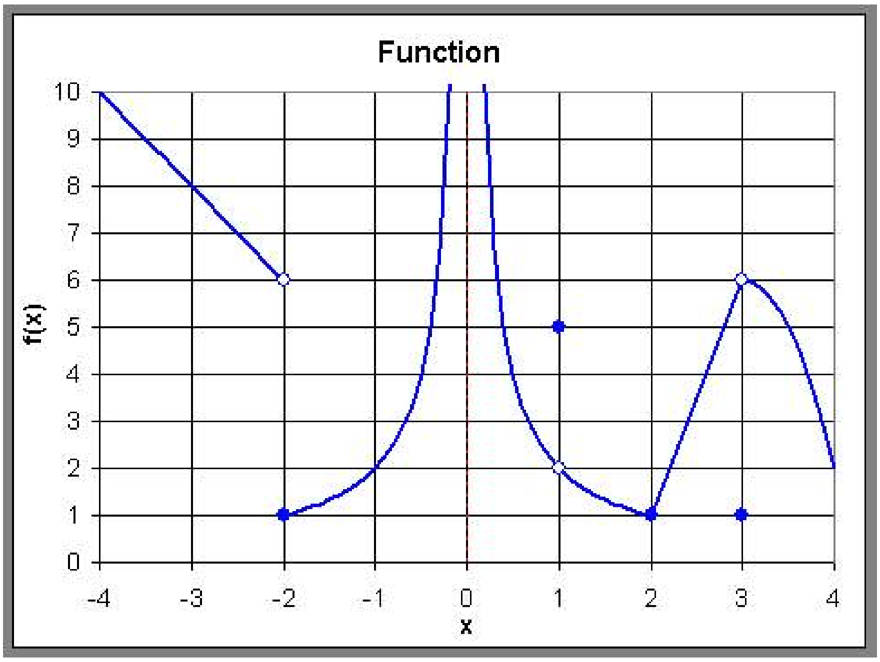
a. $f(x) = \sqrt{9-3x}$, b. $f(x) = \dfrac{x}{x+2}$,

16. Differentiate the following functions:

a. $f(x) = 4\,x^5 - 2\,x + 4 - \dfrac{2}{x^3}$,

b. $g(t) = 12\,t^{1.2} - 5\,t^3 - \dfrac{4}{\sqrt{t}} - 7$,

17. Below is a graph of a function. What is the domain of this function? At each of the integer values $-4 < x < 4$, determine the value of the function (if it exists) and find the limit at these integers. Where is this function continuous?



18. For each of the following functions, give the domain. Find all x and y-intercepts and any asymptotes, if they exist. Find the derivative of the functions, then determine any maxima or minima. Give both the x and y values. Write the second derivative and find any points of inflection. Finally, sketch the graph of the function.

a. $y = 27x - x^3$,

b. $y = x^4 - 4x$,

c. $y = x^3 + 3x^2 + 3x + 1$,

d. $y = 18x^2 - x^4$,

e. $y = x + \frac{4}{x}$.

19. Body temperatures of animals undergo circadian rhythms. A subject's temperature is measured from 8 AM until midnight, and his body temperature, T (in °C), is best approximated by the cubic polynomial

$$T(t) = 0.002(t^3 - 45t^2 + 600t + 16000),$$

where t is in hours.

a. Find the rate of change in body temperature $\frac{dT}{dt}$. What is the rate of change in body temperature at noon $t = 12$?

b. Use the derivative to find when the maximum temperature of the subject occurs and when the minimum temperature of the subject occurs. What are the body temperatures at those times? State the intervals of time where the subject's body temperature is decreasing.

20. Over a 7 day period in the summer, data were collected on an algal bloom in the ocean. The population of algae (in thousand/cc), $P(t)$, were best fit by the cubic polynomial

$$P(t) = t^3 - 9t^2 + 15t + 30,$$

where t is in days.

a. Find the rate of change in population per day, $\frac{dP}{dt}$. What is the rate of change in the population on the first day, $t = 2$?

b. Use the derivative to find when the relative minimum and maximum populations of algae occur over the time of the survey. Give the populations at those times. Over what intervals of time is the population increasing?

c. Sketch a graph of this polynomial fit to the population of algae. Show clearly the maximum and minimum populations on your graph and include the populations at the beginning of the survey ($t = 0$) and at the end ($t = 7$).