1. a. A population of herbivores satisfies the growth equation

$$H_{n+1} = 1.02H_n.$$

If the initial population is $H_0 = 2000$, then determine the populations H_1 and H_2. Also, give an expression for the population H_n in terms of H_0 and n.

b. Another group of herbivores satisfies the growth equation

$$G_{n+1} = 1.03G_n.$$

If the initial population is $G_0 = 200$, then give an expression for the population G_n in terms of G_0 and n. Determine how long does it take for this population to double.

c. Find when the two populations are equal.

2. The population of the United States was about 179.3 million in 1960 and 226.5 million in 1980. Let 1960 be represented by P_0 and assume that its population is growing according to the Malthusian growth law,

$$P_{n+1} = (1 + r)P_n,$$

where n is in years.

a. Use the data above to find the annual growth rate r, then write an expression for the population in any year following 1960. (Write the solution P_n in terms of P_0 with n being the number of years after 1960.)

b. Predict the population in the year 2000. The actual population was about 281.4 million. What is the error between the model and the actual census data?

c. According to the model, how long until the U. S. population doubles from its 1960 level?

3. a. The population of the France in 1980 was about 53.9 million, and a census in 1990 showed that the population had grown to 56.7 million. Assume that this population grows according to the Malthusian growth law,

$$P_{n+1} = (1 + r)P_n,$$

where n is the number of decades after 1980, and P_n is population n decades after 1980. Use the data above to find the growth constant r, then write the general solution P_n.

b. Predict the population in the years 2000 and 2020. France's population in 2000 was 59.4 million. Use this information to compute the percent error between the Malthusian growth model and the actual census data.

c. In 1980, the population of Kenya was 16.7 million, while in 1990, it had grown to 24.2 million. Assume its population is also growing according to a Malthusian growth law. Find its rate of growth per decade and predict its population in 2000 and 2020. How long does it take for Kenya's population to double?

d. If these countries continue to grow according to these Malthusian growth laws, then determine the first year when Kenya's population will exceed that of the France and determine their populations at that time.

e. Find the annual growth rate for both France and Kenya between 1980 and 1990.

4. a. Consider a model with immigration given by

$$p_{n+1} = 0.8p_n + 300,$$

with an initial population of $p_0 = 500$. Determine the populations at the next three time intervals, p_1, p_2, and p_3.

b. Find all equilibria and determine the stability of these equilibria.

5. a. A population of herbivores reproduces annually and satisfies the discrete dynamical system:

$$P_{n+1} = P_n + rP_n,$$

where $r = 0.1$ is the net growth rate. If the initial population is $P_0 = 100$ individuals, then determine how many herbivores there are in each of the next two years. How long does it take for this population to double?

b. Often there are crowding effects due to limited resources. This is often modeled by the discrete logistic growth model. Assume these herbivores satisfy the model:

$$P_{n+1} = 1.1P_n - 0.0005P_n^2.$$

If the initial population is $P_0 = 100$, then find P_1 and P_2.

c. Find both equilibria for the discrete logistic equation in Part b.

6. A man with a chronic lung problem has a tidal volume, V_i, of 300 ml. For this experiment, Helium, He, is used to determine the functional reserve capacity, V_r. (Note that $V_r = (1-q)V_i/q$.) The mathematical model gives

$$c_{n+1} = (1-q)c_n + q\gamma,$$

where $\gamma = 5.2$ ppm.

a. The man is given an enriched mixture of air to breathe that contains 50 ppm of He. Experimentally, the concentration of He in the first two measured breaths after breathing the enriched mixture are given by $c_0 = 50$ and $c_1 = 44.6$ ppm. Use c_0 and c_1 to find q, then determine the functional reserve capacity, V_r.

b. Use your model to find the expected concentration of Helium in this patient's 3^{rd} breath, c_3. What is the equilibrium concentration of Helium in the patient's lungs? What is the stability of this equilibrium concentration?

7. Below are data on the population of insect pests living in a survey area. The insect reproduces according to a Malthusian growth model and disperses (emigrates) to surrounding regions at a constant rate. The population model for this insect pest is given by

$$P_{n+1} = (1+r)P_n - \mu,$$

where r is the rate of growth (per week) and μ is the number of pests dispersing each week to surrounding regions.

a. From the data below determine the updating function for this population, *i.e.*, find r and μ. Then use this updating function to find the population of the insect pests for weeks 3 and 4.

b. Find all equilibria for this model. Based on your iterations in Part a, what is the stability of the equilibria? (If a solution moves closer to an equilibrium point, then it is probably stable. If it moves away, then it is most likely unstable.)

c. Graph the updating function along with the identity map, $P_{n+1} = P_n$. Determine all points of intersection.

Week	Insects
0	500
1	630
2	825

8. A woman with a chronic lung problem breathes a supply of air enriched with Helium. Experimental measurements show the following concentrations of the exhaled air after she resumes normal breathing in the room.

Breath Number	0	1	2
Conc. of He (ppm)	400	352	310

The concentration of Helium in the room, γ, is not known, but assumed to be constant.

a. Assume a breathing model of the form:

$$c_{n+1} = (1 - q)c_n + q\gamma.$$

Use the data above to find the constants q, the fraction of air exchanged, and γ, the ambient concentration of Helium. Then determine the concentration of Helium in the next two breaths, c_3 and c_4.

b. Find the equilibrium concentration of Helium in the subject's lungs based on this breathing model. What is the stability of this equilibrium concentration? (If a solution moves closer to an equilibrium point, then it is probably stable. If it moves away, then it is most likely unstable.)

c. Graph the updating function along with the identity map, $c_{n+1} = c_n$, showing all intercepts ($c_n \geq 0$) and points of intersection.

9. a. An ecological survey for a particular invasive species of fish is conducted at a lake. This species reproduces in the lake and is predated upon by other fish in the lake. There is also a constant immigration of the fish from a river feeding the lake. The survey finds that the population of this fish satisfies the discrete linear growth model

$$F_{n+1} = 0.86\,F_n + 280, \qquad F_0 = 100,$$

where F_n is the population of fish in the n^{th} year and $F_0 = 100$ is the initial population surveyed. Find the populations F_1 and F_2. Find the equilibrium population, F_e. Is this a stable or unstable population?

b. The discrete model above is approximated very well by the continuous model

$$F(t) = 2000 - 1900\,e^{-0.15t}.$$

Find the F-intercept and any horizonal asymptotes for $F(t)$, then sketch a graph of the function.

c. Use $F(t)$ to find the slope of the secant line between $t = 5$ and $t = 6$. Also, find the slope of the secant line between $t = 5$ and $t = 5.1$.

d. A good approximation to the slope of the tangent line (which approximates the derivative) is found from the slope of the secant line between $t = 5$ and $t = 5.001$. Find this slope and write the equation for the tangent line. (**Warning:** Keep 8-10 significant figures on your calculator.)

10. A ball, which is thrown vertically upward with a velocity of 48 ft/sec and falls under the influence of gravity without air resistance from a 160 ft cliff, satisfies the equation

$$h(t) = 160 + 48t - 16t^2,$$

where h is in feet and t is in seconds.

a. Find the average velocity of the ball between the times $t = 0$ and $t = 2$. Also, find the average velocity between the times $t = 1$ and $t = 1.2$ and between the times $t = 1$ and $t = 1.01$

b. Sketch a graph of $h(t)$, showing crucial points, including the h-intercept, the maximum height, and when the ball hits the ground. Approximate the velocity with which the ball hits the ground by finding the average velocity of the ball between the time the ball hits the ground and 0.001 sec before it hits the ground.

11. Fish continue to grow throughout their life, but their growth slows with age. The leopard shark (*Triakis semifiate*) is a common ovoviviparous, benthic shark in the San Diego waters. A good approximation to the growth of the leopard shark uses the von Bertalanffy equation for fish growth and is given by

$$L(t) = 2.1 - 1.9e^{-0.25t},$$

where L is in meters and t is in years.

a. Determine any asymptotes. Find the approximate length of a leopard shark at birth and ages 1, 5, and 10 years. Sketch a graph of the length of a leopard shark. What is the maximum length of a leopard shark and at what age does it reach 90% of that length?

b. Find the average rate of growth between the ages of 1 and 5 years, between the ages of 5 and 10 years, between the ages of 5 and 6 years, and between the ages of 5 and 5.01 years. Which of these gives the best approximation to the derivative (instanteous rate of growth) at $t = 5$ years?

12. a. A serval is a 20-40 pound wild cat that has an incredible ability to leap and catch birds. Its predation success rate ranks among the highest of any animals. Suppose that a serval is sitting on a ledge 16 ft above the ground and can jump with a vertical velocity of 24 ft/sec. If we ignore air resistance and use an acceleration from gravity of -32 ft/sec^2, then the height of the serval above the ground, $h(t)$, is given by the formula

$$h(t) = 16 + 24t - 16t^2.$$

Find the range of heights for a bird flying above the serval that allows this predator a chance to catch it.

b. Find the average velocity of the serval for the intervals $t \in [0, \frac{1}{4}]$, $t \in [\frac{1}{2}, 1]$, and $t \in [1, \frac{5}{4}]$.

c. Find the instantaneous velocity at $t = 1$ by computing

$$v(1) = \frac{h(1 + \Delta t) - h(1)}{\Delta t},$$

and finding the limiting value for $\Delta t \to 0$.

d. Determine the time when the serval hits the ground. Sketch a graph of the height of the serval as a function of t.

13. An impala is migrating across a field that has been fenced with a 180 cm fence. To escape it needs to jump this fence. Assume that the impala jumps the fence with just enough vertical velocity, v_0 to clear it. If the height (in cm) of the impala is given by

$$h(t) = v_0 t - 490t^2.$$

a. Use the height of the fence (maximum height) and the function describing the height of the impala, $h(t)$, to determine the vertical velocity, v_0, then determine how long the impala is in the air.

b. With this value of v_0, find the average velocity of the impala between $t = 0$ and $t = 0.5$.

14. a. Find the slope of the secant line through the points $(2, f(2))$ and $(2+\Delta x, f(2+\Delta x))$ for the function

$$f(x) = 2x - x^2.$$

b. Let Δx get small and determine the slope of the tangent line through $(2, f(2))$, which gives the value of the derivative of $f(x)$ at $x = 2$. Give the equation of the tangent line.

15. Consider the function

$$f(x) = \frac{2}{3 - x}.$$

a. Sketch a graph of the function, including any asymptotes.

b. Find the slope of the secant line through the points $(2, f(2))$ and $(2+\Delta x, f(2+\Delta x))$ for the function

c. Let Δx get small and determine the slope of the tangent line through $(2, f(2))$, which gives the value of the derivative of $f(x)$ at $x = 2$. Give the equation of this tangent line.

16. Consider the function

$$f(x) = \sqrt{9 - 5x}.$$

a. Sketch a graph of the function.

b. Compute the slope of the secant line through the points $x = 1$ and $x = 1 + \Delta x$.

c. From your answer in Part b., take the limit as $\Delta x \to 0$ to obtain the slope of the tangent line, then write the equation of the tangent line at $x = 1$.

17. Use the definition of the derivative to find the derivative of the following functions.

a. $f(x) = \sqrt{9 - 3x}$,

b. $f(x) = \dfrac{x}{x+2}$,

18. Differentiate the following functions:

a. $f(x) = 4\,x^5 - 2\,x + 4 - \dfrac{2}{x^3}$,

b. $g(t) = 12\,t^{1.2} - 5\,t^3 - \dfrac{4}{\sqrt{t}} - 7$,

19. Below is a graph of a function. What is the domain of this function? At each of the integer values $-4 < x < 4$, determine the value of the function (if it exists) and find the limit at these integers. Where is this function continuous?