1. Consider the points $(0, -3)$ and $(4, -1)$. Find the equation of the line (in point-slope form) passing through the two points. Also, find the equation of the line perpendicular to this line passing through the second of these points. Sketch a graph of these two lines.

2. Suppose that a 43 lb dog has a temperature of 102°F. Write the weight and temperature of this dog in kilograms and degrees Celsius. (Note that 1 kg = 2.2046 lb.)

3. Consider the functions:

$$f(x) = 2x - 1 \qquad \text{and} \qquad g(x) = 15 + 2x - x^2.$$

Find the coordinates of the x and y-intercepts for both functions. Find the slope of the line and the coordinates for the vertex of the parabola. Determine the coordinates for all points of intersection and sketch the graph.

4. Consider the functions $f(x) = x - 3$ and $g(x) = x^2 - 4x - 3$. Sketch the graphs of these functions. Include the coordinates of x and y-intercepts for both functions and the vertex of the parabola. Find the points of intersection.

5. The table below shows evaporation of water from a beaker. Initially, there is one liter. The loss by evaporation is linear. Find the equation of the line for V as a function of t. Determine when all the water is lost. Graph this function for all t when there is water in the beaker.

Time (t)	Volume (V)
0 week	1000 ml
1 week	940 ml
2 week	880 ml
4 week	760 ml

6. The following growth data were recorded for the height of a plant.

Week (t)	0	1	2	4
Height (cm) (h)	10.5	14	15.5	18

A linear model is proposed for the growth of this plant and has the form

$$h(t) = 2t + 11.$$

a. Find the sum of squares error between the data and the model. Sketch a graph of the model with the data points. Is the model reasonable?

b. Use the model to predict the height of the plant at 3 and 5 weeks.

7. For animals that reproduce seasonally, we find that their population satisfies a difference equation

$$P_{n+1} = P_n + g(P_n),$$

where P_n is the population in the nth season and $g(P)$ (in individuals per generation) is the growth rate of the population.

a. Suppose that the growth rate $g(P)$ satisfies the quadratic equation

$$g(P) = 0.1P\left(1 - \frac{P}{400}\right).$$

The population is at equilibrium when the growth rate is zero. Find the equilibrium populations.

b. The growth rate is at a maximum at the vertex of parabola. Find the population that produces this maximum growth rate and what that growth rate is.

c. Sketch a graph of this growth rate function.

8. If i $(0 \leq i \leq 1)$ is the fraction of infectious people in a community with an air-borne disease that imparts no immunity, then the fraction of susceptible people is $1 - i$. Assume that new infectious people are added at a rate $\alpha i(1 - i)$ with $\alpha = 0.1$ and infectious people are cured at a rate βi with $\beta = 0.07$. The rate of change in infectious people satisfies the function:

$$F(i) = 0.1i(1 - i) - 0.07i.$$

a. The disease is at equilibrium when the infection rate, $F(i)$, is zero. Find the equilibrium fractions of infectious people for this disease. (Hint: First rewrite $F(i)$ into standard factored quadratic form.)

b. The rate of spreading of the disease is at a maximum at the vertex of parabola. Find the fraction of infectious people, i, that produces this maximum rate of infection and what that infection rate is.

c. Find $F(1)$, then sketch a graph of this infection rate function, $F(i), \quad 0 \leq i \leq 1$.

9. A rectangle with a length x and width y has a perimeter of 20 cm.

a. Write an expression for the width y as a function of the length x, using this information.

b. The area of a rectangle is $A = xy$. Substitute the expression for y into this formula for the area to produce a function of the area as a function of x alone. What is the domain of this function?

c. Sketch a graph of the area as a function of x and determine what value of x produces the largest area. What curve is produced by $A(x)$?

10. Suppose that $e^a = 2.2$ and $e^b = 0.7$. In addition, assume that $\ln(c) = 1.3$ and $\ln(d) = -0.5$. Use the properties of exponentials and logarithms to evaluate the following:

a. $\dfrac{e^{a+b}}{(e^b + e^0)^2}$

b. $\dfrac{(e^a)^2(e^0 - e^b)}{e^{2a+b}}$

c. $\dfrac{(\ln(c^3) - \ln(c) + \ln(1))}{(\ln(c) + \ln(e))}$,

d. $\dfrac{\ln(c^2 d) - \ln(1))}{(\ln(c/d) - \ln(e))}$

11. For each of the following functions, give the domain. Find the x and y-intercepts, and determine all vertical and horizontal asymptotes for each of these functions, then sketch a graph.

a. $y = x^3 - x^2 - 12x$

b. $y = \dfrac{50}{25 - x^2}$

c. $y = \dfrac{6x}{x+2}$

d. $y = \sqrt{16 - 2x}$

e. $y = 3x - 2x^2 - x^3$,

f. $y = 4 - \sqrt{5 - x}$,

g. $y = 20 - 5e^{-0.5x}$,

h. $y = 6\ln(5 - x) - 2$,

i. $y = \dfrac{4x}{2 + 0.001x}$,

j. $y = \dfrac{8x}{4 - x^2}$.

k. $y = 3 + 2\ln(x + 1)$,

l. $y = 6e^{x/2} - 2$,

12. You are given the following data on the heights and lengths of several breeds of dogs. The height is measured at the shoulder (in cm) and the length is from the nose to the anus (in cm).

Breed	Height (H cm)	Length (L cm)
Chihuahua	18	33
Beagle	36	81
Labrador Retriever	55	102
Irish Setter	66	115

A linear model is proposed for the relationship between the length, L, and the height, H, of the following form:

$$L(H) = 1.7H + 10$$

a. Find the sum of squares error between the data and the model. Which breed is furthest from the model?

b. Use the model to predict the length of a Borzoi, which has a shoulder height of 81 cm. Also, use the model to predict the height of a Border Collie, which has a length of 85 cm.

13. The Lambert-Beer law for absorbance of light by a spectrophotometer is a linear relationship, which can have the form

$$A = mc,$$

where c is the concentration of the sample, A is absorbance, and m is the slope that must be determined from experiments.

a. Below are data collected on samples from a collection of urea standards using a urea indicator.

c (mM)	1	3	6
A	0.5	1.7	3.2

Write a formula for the quadratic function $J(m)$ that measures the sum of squares error of the line fitting the data. Find the vertex of this quadratic function. This gives the value of the best slope m, while the $J(m)$ value of the vertex gives the least sum of squares error.

b. Use this model (with the best value of m) to determine the concentration of urea in an unknown sample with absorbances of $A = 2.2$.

14. An underwater ecological study is made easier by photographing the region, then measuring distances on the picture. The picture is taken from above a flat rock reef. The diver measures three reference objects to help him with his study. One distinctive rock is 1.2 meters and measures 2.0 cm on the picture. Two kelp plants are separated by 2.5 meters, which is 4.0 cm on the picture. A sand bar is 3.6 meters across and measures 6.1 cm in the picture.

a. The conversion of measurements in the photo p to measurements in actual distance d is given by the formula

$$d = kp.$$

Write a formula for the quadratic function $J(k)$ that measures the sum of squares error of the line fitting the measurements in the photo. Find the vertex of this quadratic function. This gives the value of the best slope k, while the $J(k)$ value of the vertex gives the least sum of squares error.

b. In the photograph, there is a picture of a leopard shark that measures 2.2 cm. How long is this shark?

c. How long would a 2.0 m shark appear in the picture?

15. The poultry industry has accumulated detailed data on the consumption of feed by chickens. The reference *Nutritional Requirement of Chickens* (1984), you are given that a 560 g chicken consumes 390 g of feed per week. A 2520 g broiler consumes 1210 g of feed per week.

a. Assume linear relationship between the weight of the chicken (W) and the amount of feed (F) that it consumes

$$F = mW + b.$$

Use the data above to find the constants m and b in the model above.

b. Assume there is a power law relationship between the weight of the chicken (W) and the amount of feed (F) that it consumes

$$F = kW^a.$$

Use the data above to find the constants a and k in the power law or allometric model above.

c. Use both models (linear and allometric) to find the amount of feed consumed by a 1000 g chicken. Also, estimate the weight of a chicken that consumes 500 g of feed using both models. Which model gives the better predictions and why?

16. Experimental measurements show that when current is applied to samples of a tissue, the resistance measured by a voltmeter yields the thickness, T. Suppose a 3 mm sample of tissue causes a voltage drop, v, of 0.25V, and a 4 mm sample of tissue causes a voltage drop of 0.45V.

a. A linear model is given by $T = mv + b$ for some constants m and b. Find the constants m and b and sketch a graph of this model. Use this model to predict the voltage drop for a tissue that has a thickness of 2 mm. Also, find the thickness of a tissue that gives a voltage drop of 0.6V.

b. If the thickness of the tissue satisfies a power law with respect to resistance measured by the voltage drop, then the model is given by

$$T = kv^a,$$

Find the constants k and a and sketch a graph of this model. Use this model to predict the voltage drop for a tissue that has a thickness of 2 mm. Also, find the thickness of a tissue that gives a voltage drop of 0.6V.

c. Which model do you expect is better and why?