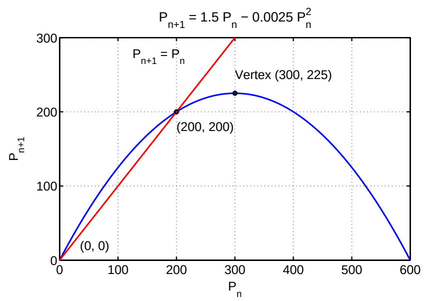
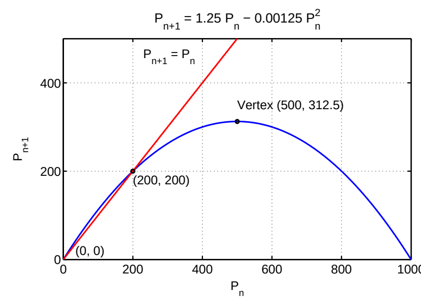
1. a. $P_1 = 68.75$, $P_2 = 91.3$, and $P_3 = 116.1$.

b. The P_n-intercepts are $(0, 0)$ and $(600, 0)$, and the vertex is $(300, 225)$. The equilibria are $P_e = 0$ and 200. Since $F'(P) = 1.5 - 0.005P$, we have $F'(0) = 1.5$, and the equilibrium at $P_e = 0$ is unstable (monotonically). Also, we have $F'(200) = 0.5$, and the equilibrium at $P_e = 200$ is stable (monotonically).



Problem 1



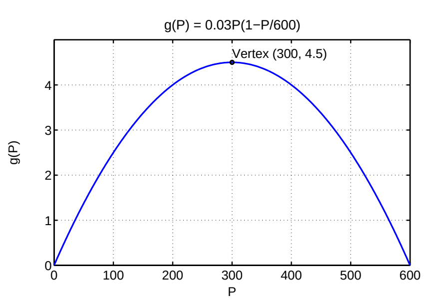
Problem 2

2. a. $P_1 = 234.375$, $P_2 = 224.3$, and $P_3 = 217.49$. The equilibria are $P_e = 0$ and 200. Since $F'(P) = 1.25 - 0.0025P$, we have $F'(0) = 1.25$, and the equilibrium at $P_e = 0$ is unstable (monotonically). Also, we have $F'(200) = 0.75$, and the equilibrium at $P_e = 200$ is stable (monotonically).

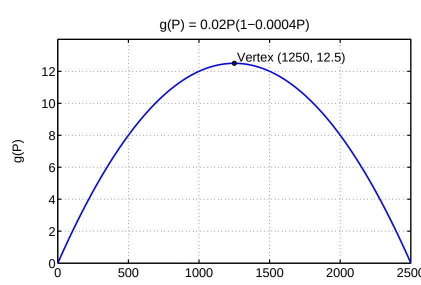
b. The P_n-intercepts are $(0, 0)$ and $(1000, 0)$, and the vertex is $(500, 312.5)$. The equilibria are $P_e = 0$ and 200. Since $F'(P) = 1.25 - 0.0025P$, we have $F'(0) = 1.25$, and the equilibrium at $P_e = 0$ is unstable (monotonically). Also, we have $F'(200) = 0.75$, and the equilibrium at $P_e = 200$ is stable (monotonically).

3. a. The growth rate is zero at $P = 0$ and 600, and the vertex is $(300, 4.5)$.

b. $P_1 = 102.5$, $P_2 = 105.05$, and $P_3 = 107.65$. The equilibria are $P_e = 0$ and 600. Since the updating function is $F(P) = P + 0.03P(1 - P/600)$, we have $F'(P) = 1.03 - 0.0001P$. Thus, we find $F'(0) = 1.03$, and the equilibrium at $P_e = 0$ is unstable (monotonically). Also, we have $F'(600) = 0.97$, and the equilibrium at $P_e = 600$ is stable (monotonically).



Problem 3



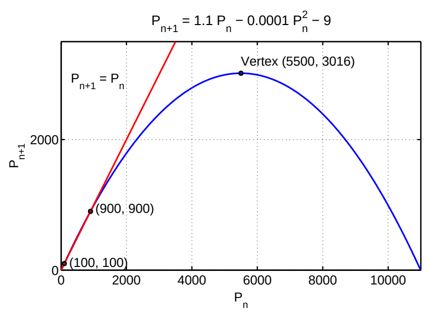
Problem 4

4. a. The growth rate is zero at $P = 0$ and 2500, and the vertex is $(1250, 12.5)$.

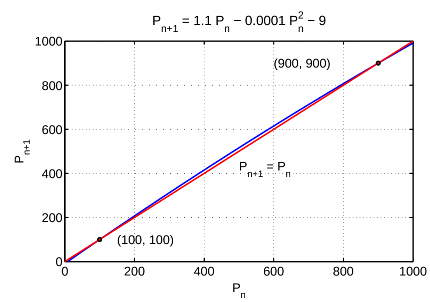
b. $P_1 = 4900$, $P_2 = 4805.92$, and $P_3 = 4717.26$. The equilibria are $P_e = 0$ and 2500. Since the updating function is $F(P) = P + 0.02P(1 - 0.0004P)$, we have $F'(P) = 1.02 - 0.000016P$. Thus, we find $F'(0) = 1.02$, and the equilibrium at $P_e = 0$ is unstable (monotonically). Also, we have $F'(600) = 0.96$, and the equilibrium at $P_e = 2500$ is stable (monotonically).

5. a. $P_1 = 516$, $P_2 = 531.97$, and $P_3 = 547.87$.

b. The P_n-intercepts are $(8.1879, 0)$ and $(10991.8, 0)$, and the vertex is $(5500, 3016)$. The equilibria are $P_e = 100$ and 900. Since $F'(P) = 1.1 - 0.0002P$, we have $F'(100) = 1.08$, and the equilibrium at $P_e = 900$ is unstable (monotonically). Also, we have $F'(900) = 0.92$, and the equilibrium at $P_e = 900$ is stable (monotonically).



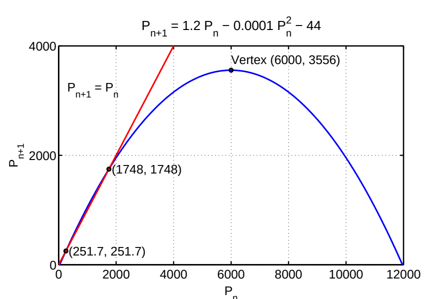
Problem 5



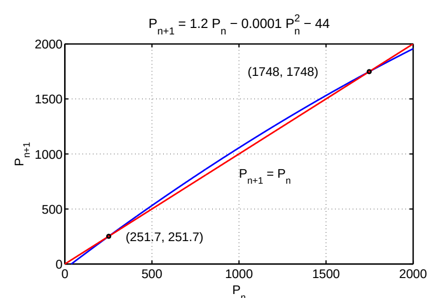
Problem 5 (close)

6. a. $P_1 = 1056$, $P_2 = 1111.7$, and $P_3 = 1166.4$.

b. The P_n-intercepts are $(36.779, 0)$ and $(11963.2, 0)$, and the vertex is $(6000, 3556)$. The equilibria are $P_e = 251.67$ and 1748.3. Since $F'(P) = 1.2 - 0.0002P$, we have $F'(251.67) = 1.1497$, and the equilibrium at $P_e = 1748.3$ is unstable (monotonically). Also, we have $F'(1748.3) = 0.8503$, and the equilibrium at $P_e = 1748.3$ is stable (monotonically).



Problem 6



Problem 6 (close)

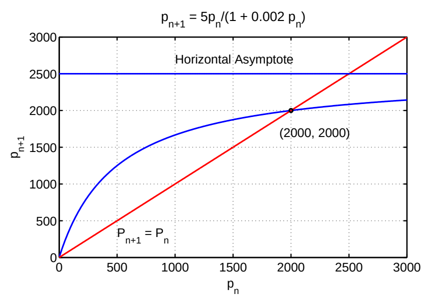
7. a. $p_1 = 1250$, $p_2 = 1785.7$, and $p_3 = 1953.1$.

b. The p_n-intercept is $(0, 0)$, and there is a horizontal asymptote at $p_{n+1} = 2500$.

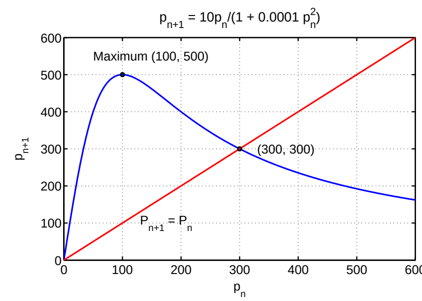
c. The equilibria are $p_e = 0$ and 2000. Since

$$B'(p) = \frac{5}{(1 + 0.002p)^2},$$

we have $B'(0) = 5$, and the equilibrium at $p_e = 0$ is unstable (monotonically). Also, we have $B'(2000) = 0.2$, and the equilibrium at $P_e = 2000$ is stable (monotonically).



Problem 7



Problem 8

8. a. $p_1 = 500$, $p_2 = 192.3$, and $p_3 = 409.3$.

b. The p_n-intercept is $(0, 0)$, and there is a horizontal asymptote at $p_{n+1} = 0$. A maximum occurs at $(100, 500)$.

c. The equilibria are $p_e = 0$ and 300. Since

$$H'(p) = \frac{10 - 0.001p^2}{(1 + 0.0001p^2)^2},$$

we have $H'(0) = 10$, and the equilibrium at $p_e = 0$ is unstable (monotonically). Also, we have $H'(300) = -0.8$, and the equilibrium at $P_e = 300$ is stable (oscillatory).

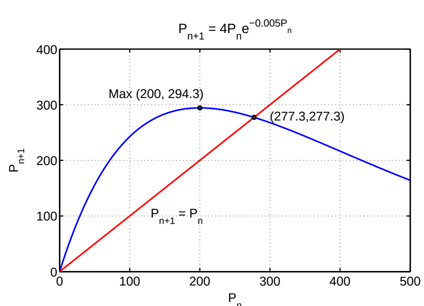
9. a. $P_1 = 242.6$, $P_2 = 288.5$, and $P_3 = 272.7$.

b. The P_n-intercept is $(0, 0)$, and there is a horizontal asymptote at $P_{n+1} = 0$. A maximum occurs at $(200, 294.3)$.

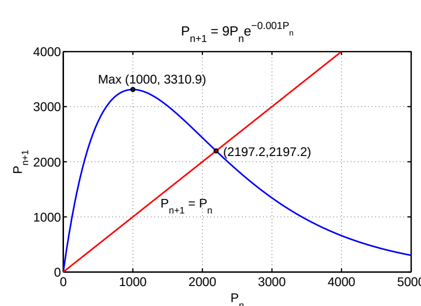
c. The equilibria are $P_e = 0$ and 277.26. Since

$$R'(P) = 4e^{-0.005P}(1 - 0.005P),$$

we have $R'(0) = 4$, and the equilibrium at $P_e = 0$ is unstable (monotonically). Also, we have $R'(277.26) = -0.3863$, and the equilibrium at $P_e = 277.26$ is stable (oscillatory).



Problem 9



Problem 10

10. a. $P_1 = 814.35$, $P_2 = 3246.3$, and $P_3 = 1137.1$.

b. The P_n-intercept is $(0,0)$, and there is a horizontal asymptote at $P_{n+1} = 0$. A maximum occurs at $(1000, 3310.9)$.

c. The equilibria are $P_e = 0$ and 2197.2. Since

$$R'(P) = 9e^{-0.001P}(1 - 0.001P),$$

we have $R'(0) = 9$, and the equilibrium at $P_e = 0$ is unstable (monotonically). Also, we have $R'(2197.2) = -1.197$, and the equilibrium at $P_e = 2197.2$ is unstable (oscillatory).

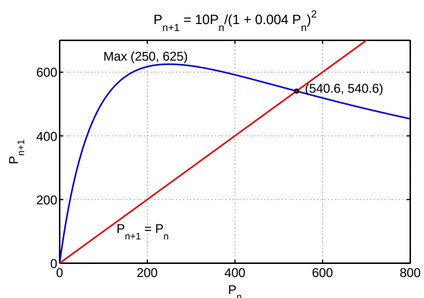
13. a. $P_1 = 510.2$, $P_2 = 551.8$, and $P_3 = 536.5$.

b. The P_n-intercept is $(0,0)$, and there is a horizontal asymptote at $P_{n+1} = 0$. A maximum occurs at $(250, 625)$.

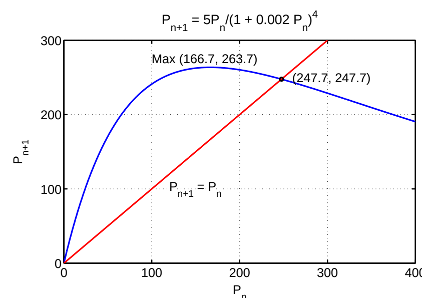
c. The equilibria are $P_e = 0$ and 540.6. Since

$$H'(P) = \frac{10 - 0.04P}{(1 + 0.004P)^3},$$

we have $H'(0) = 10$, and the equilibrium at $P_e = 0$ is unstable (monotonically). Also, we have $H'(540.6) = -0.3675$, and the equilibrium at $P_e = 0$ is stable (oscillatory).



Problem 13



Problem 14

14. a. $P_1 = 357.1$, $P_2 = 735.3$, and $P_3 = 932.8$.

b. The P_n-intercept is $(0,0)$, and there is a horizontal asymptote at $P_{n+1} = 1250$.

c. The equilibria are $P_e = 0$ and 1000. Since

$$B'(P) = \frac{5}{(1 + 0.004P)^2},$$

we have $B'(0) = 5$, and the equilibrium at $P_e = 0$ is unstable (monotonically). Also, we have $B'(1000) = 0.2$, and the equilibrium at $P_e = 1000$ is stable (monotonic).

15. a. $N_1 = 4.6$, $N_2 = 5.4$, and $N_3 = 6.465$.

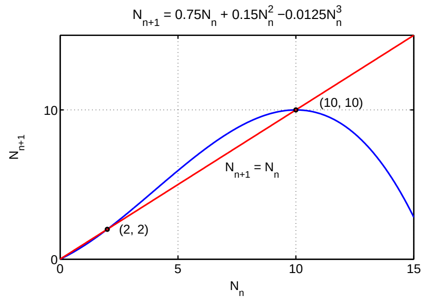
b. The equilibria for this cubic equation are $N_e = 0$, 2, and 10.

c. The derivative of the updating function is

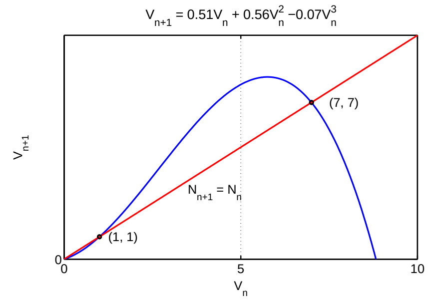
$$A'(N) = \frac{3}{4} + \frac{3N}{10} - \frac{3N^2}{80}.$$

Thus, we have $A'(0) = 3/4$, and the equilibrium at $N_e = 0$ is stable (monotonically). Also, we have $A'(2) = 1.2$, and the equilibrium at $N_e = 2$ is unstable (monotonic). Finally, the equilibrium at $N_e = 10$ has $A'(10) = 0$, and this equilibrium at $N_e = 10$ is stable (monotonic).

d. This Allee effect shows that if the flamingo population falls below $N_e = 2$ (thousand), then the population will tend toward zero (extinction). If the population is above $N_e = 2$ (thousand), then the population will tend toward the carrying capacity of $N_e = 10$ (thousand).



Problem 15



Problem 16

16. a. $V_1 = 4.68$, $V_2 = 7.48$, and $V_3 = 5.86$.

b. The equilibria for this cubic equation are $V_e = 0$, 1, and 7.

c. The derivative of the updating function is

$$M'(V) = 0.51 + 1.12V - 0.21V^3.$$

Thus, we have $M'(0) = 0.51$, so the equilibrium at $V_e = 0$ is stable (monotonically). Also, we have $M'(1) = 1.42$, so the equilibrium at $V_e = 1$ is unstable (monotonic). Finally, the equilibrium at $V_e = 7$ has $M'(7) = -1.94$, so this equilibrium at $V_e = 7$ is unstable (oscillatory).

d. The dynamical model for a nerve cell shows that a small stimulus ($V_0 < 1$) will return to rest with $V_e = 0$. When the stimulus is larger ($V_0 > 1$), then the nerve cell will fire continuously in an oscillatory manner with the voltage going above and below the unstable active equilibrium $V_e = 7$.