1. Consider the discrete logistic growth model given by

$$P_{n+1} = 1.5P_n - 0.0025P_n^2.$$

a. Suppose that the initial population P_0 is 50. Find the population of the next three generations, P_1, P_2, and P_3.

b. Sketch a graph of the updating function with the identity map, $P_{n+1} = P_n$. Be sure to show the intercepts of the parabola as well as the vertex. Find the equilibria and identify them on your graph.

c. Find the derivative of the updating function, then use the results from the lecture notes to determine the behavior near each of the equilibria.

2. Consider the discrete logistic growth model given by

$$P_{n+1} = f(P_n) = 2.25P_n - 0.00125P_n^2.$$

a. Suppose that the initial population $P_0 = 2000$. Find the population of the next three generations, P_1, P_2, and P_3. Find all equilibria.

b. Sketch a graph of the updating function, $f(P)$, with the identity map, $P_{n+1} = P_n$. Find the intercepts and the vertex of the parabola. Find the equilibria and identify them on your graph.

c. Find the derivative of $f(P)$, then use the results from the lecture notes to determine the behavior near each of the equilibria.

3. Assume that the growth rate of a population P satisfies

$$g(P) = 0.03P(1 - P/600).$$

The discrete logistic growth model for this population is given by:

$$P_{n+1} = P_n + g(P_n).$$

a. Find the population when the growth rate $g(P)$ is zero (the P-intercepts) and when it is a maximum (the vertex). Sketch the graph of $g(P)$.

b. Find all equilibria, then determine the behavior of the model near the equilibria.

4. Assume that the growth rate of a population P satisfies

$$g(P) = 0.02P(1 - 0.0004P).$$

The discrete logistic growth model for this population is given by:

$$P_{n+1} = P_n + g(P_n).$$

a. Find the population when the growth rate $g(P)$ is zero and when it is a maximum. Sketch the graph of $g(P)$.

b. Find all equilibria, then determine the behavior of the model near the equilibria.

5. A modified version of the discrete logistic growth model that includes emigration is given by

$$P_{n+1} = f(P_n) = 1.1P_n - 0.0001P_n^2 - 9.$$

a. Suppose that the initial population P_0 is 500. Find the population of the next three generations, P_1, P_2, and P_3.

b. Sketch a graph of the updating function with the identity map, $P_{n+1} = P_n$. Be sure to show the intercepts of the parabola as well as the vertex. Find the equilibria and identify them on your graph.

c. Determine the behavior of the solution to this modified logistic growth model near the equilibria.

6. Consider the Logistic growth model given by the discrete dynamical model

$$P_{n+1} = F(P_n) = 3.1P_n - 0.0002P_n^2,$$

where P_n is the population after n generations.

a. Suppose that initially there are 1000 individuals, so $P_0 = 1000$. Find the populations at the end of the first three generations P_1, P_2, and P_3.

b. Find the equilibria for this model, then use the derivative of the updating function, $F'(P)$, to determine the behavior of the solution near the equilibrium.

c. Sketch the updating function and the identity function ($P_{n+1} = P_n$), showing the vertex of $F(P)$, the points of intersection, and any intercepts.

7. a. A population of yeast is growing according to the Malthusian growth model

$$P_{n+1} = (1 + r)P_n, \quad \text{with} \quad P_0 = 15,$$

where P_0 is the initial population (in 1000/cc) and n is in hours. The population after two hours is found to be $P_2 = 21$ (in 1000/cc). Find the value of r (to **4 significant figures**), then determine how long it takes for this population to double.

b. After a few hours, the nutrient supply becomes limiting for this culture, so a Logistic growth model better describes the population of yeast. Suppose that experiments show the population follows the model given by

$$P_{n+1} = F(P_n) = 1.22P_n - 0.0004P_n^2,$$

where again n is in hours and $P_0 = 15$ (in 1000/cc). Find the equilibria for this Logistic growth model. Calculate the derivative of $F(P)$. Determine the behavior of the solution near the equilibria by giving the value of the derivative for each equilibrium. Is the solution stable or unstable near each equilibrium? Also, is the solution monotonic or does it oscillate?

c. Sketch the updating function, $F(P)$, and the identity function ($P_{n+1} = P_n$), showing the vertex of $F(P)$, the points of intersection, and any intercepts.

8. a. The population of France in 1950 was about 41.8 million, while in 1970, it was about 50.8 million. Assume that the population is growing according to the discrete Malthusian growth equation

$$P_{n+1} = (1 + r)P_n, \quad \text{with} \quad P_0 = 41.8,$$

where P_0 is the population in 1950 and n is in decades. Use the population in 1970 (P_2) to find the value of r (to 4 significant figures). Write the formula for the general solution to this model.

b. Estimate the population in 2000 based on this model. Given that the population in 2000 was 59.4million, find the percent error between the actual and predicted values.

c. A better model fitting the census data for France is a Logistic growth model given by

$$P_{n+1} = F(P_n) = 1.28P_n - 0.00416P_n^2,$$

where again n is in decades after 1950. If $P_0 = 41.8$, then use this model to predict the populations in 1960 and 1970.

d. Find the equilibria for this Logistic growth model. Calculate the derivative of $F(P)$ and evaluate it at the larger of the equilibria. What does this value say about the behavior of the solution near this equilibrium?