1. Consider the discrete logistic growth model given by

$$P_{n+1} = 1.5\,P_n - 0.0025\,P_n^2.$$

a. Suppose that the initial population P_0 is 50. Find the population of the next three generations, P_1, P_2, and P_3.

b. Sketch a graph of the updating function with the identity map, $P_{n+1} = P_n$. Be sure to show the intercepts of the parabola as well as the vertex. Find the equilibria and identify them with your graph. What is the stability of the equilibria?

2. Consider the discrete logistic growth model given by

$$P_{n+1} = f(P_n) = 1.25P_n - 0.00125P_n^2.$$

a. Suppose that the initial population $P_0 = 250$. Find the population of the next three generations, P_1, P_2, and P_3. Find all equilibria. Determine the stability of the equilibria.

b. Sketch a graph of the updating function, $f(P)$, with the identity map, $P_{n+1} = P_n$. Find the intercepts and the vertex of the parabola.

3. Assume that the growth rate of a population P satisfies

$$g(P) = 0.03P(1 - P/600).$$

The discrete logistic growth model for this population is given by:

$$P_{n+1} = P_n + g(P_n).$$

a. Find the population when the growth rate $g(P)$ is zero (the P-intercepts) and when it is a maximum (the vertex). Sketch the graph of $g(P)$.

b. Let $P_0 = 100$ and compute P_1, P_2, and P_3. Find all equilibria. Determine the stability of the equilibria.

4. Assume that the growth rate of a population P satisfies

$$g(P) = 0.02P(1 - 0.0004P).$$

The discrete logistic growth model for this population is given by:

$$P_{n+1} = P_n + g(P_n).$$

a. Find the population when the growth rate $g(P)$ is zero and when it is a maximum. Sketch the graph of $g(P)$.

b. Let $P_0 = 5000$ and compute P_1, P_2, and P_3. Find all equilibria. Determine the stability of the equilibria.

5. A modified version of the discrete logistic growth model that includes emigration is given by

$$P_{n+1} = f(P_n) = 1.1P_n - 0.0001P_n^2 - 9.$$

a. Suppose that the initial population P_0 is 500. Find the population of the next three generations, P_1, P_2, and P_3.

b. Sketch a graph of the updating function with the identity map, $P_{n+1} = P_n$. Be sure to show the intercepts of the parabola as well as the vertex. Find the equilibria and identify them on your graph. Determine the stability of the equilibria.

6. A modified version of the discrete logistic growth model that includes immigration is given by

$$p_{n+1} = f(p_n) = 1.2p_n - 0.0001p_n^2 - 44.$$

a. Suppose that the initial population $P_0 = 1000$. Find the population of the next three generations, p_1, p_2, and p_3.

b. Sketch a graph of the updating function with the identity map, $p_{n+1} = p_n$. Be sure to show the intercepts of the parabola as well as the vertex. Find the equilibria and identify them on your graph. Determine the stability of the equilibria.

7. Another common model used in ecology is the Beverton-Holt model. Consider the model that is given by

$$p_{n+1} = B(p_n) = \frac{5p_n}{1 + 0.002p_n}.$$

a. Assume that $p_0 = 500$ and find the population for the next three generations, p_1, p_2, and p_3.

b. Find the p-intercepts and the horizontal asymptote for $B(p)$ and sketch a graph of $B(p)$ for $p > 0$ along with the identity map, $p_{n+1} = p_n$.

c. By solving $p_e = B(p_e)$, determine all equilibria for this model. Find the stability of the equilibria.

8. Hassell modified the Beverton-Holt model with a power in the denominator. Consider Hassell's model that is given by

$$p_{n+1} = H(p_n) = \frac{10p_n}{1 + 0.0001p_n^2}.$$

a. Assume that $p_0 = 100$ and find the population for the next three generations, p_1, p_2, and p_3.

b. Find the p-intercepts and the horizontal asymptote for $H(p)$ and sketch a graph of $H(p)$ for $p > 0$ along with the identity map, $p_{n+1} = p_n$. Find all extrema and show them on the graph.

c. By solving $p_e = H(p_e)$, determine all equilibria for this model. Find the stability of the equilibria.

9. Many biologists in fishery management use Ricker's model to study the population of fish. Let P_n be the population of fish in any year n, then Ricker's model is given by

$$P_{n+1} = R(P_n) = aP_n e^{-bP_n}.$$

Suppose that the best fit to a set of data gives $a = 4$ and $b = 0.005$ for the number of fish sampled from a particular river.

a. Let $P_0 = 100$, then find P_1, P_2, and P_3.

b. Sketch a graph of R(P) with the identity function, showing the intercepts, all extrema, and any asymptotes.

c. Find all equilibria of the model and describe the behavior of these equilibria.

10. Repeat Exercise 1 with $a = 9$ and $b = 0.001$

11. Consider the chalone model for mitosis given by the equation

$$P_{n+1} = f(P_n) = \frac{2P_n}{1 + (bP_n)^c},$$

where $b = 0.05$ and $c = 2$.

a. Let $P_0 = 10$, then find P_1, P_2, and P_3.

b. Sketch a graph of $f(P)$ with the identity function for $P \geq 0$, showing the intercepts, all extrema, and any asymptotes.

c. Find all equilibria of the model and describe the behavior of these equilibria.

12. Some entomologists use the Beverton-Holt model for studying the population of insects. Let P_n be the population of a species of beetle in week n and suppose that the Beverton-Holt model is given by

$$P_{n+1} = B(P_n) = \frac{aP_n}{1 + bP_n}.$$

Suppose that the best fit to a set of data gives $a = 5$ and $b = 0.004$ for this species of beetle.

a. Let $P_0 = 100$, then find P_1, P_2, and P_3.

b. Sketch a graph of $B(P)$ with the identity function for $P \geq 0$, showing the intercepts and any asymptotes.

c. Find all equilibria of the model and describe the behavior of these equilibria.

13. The general form of Hassell's model is used to study a population of insects. Let P_n be the population of a species of moth in week n and suppose that Hassell's model is given by

$$P_{n+1} = H(P_n) = \frac{aP_n}{(1 + bP_n)^c}.$$

Suppose that the best fit to a set of data gives $a = 10$, $b = 0.004$, and $c = 2$. for this species of moth.

a. Let $P_0 = 100$, then find P_1, P_2, and P_3.

b. Sketch a graph of $H(P)$ with the identity function for $P \geq 0$, showing the intercepts, all extrema, and any asymptotes.

c. Find all equilibria of the model and describe the behavior of these equilibria.

14. Repeat the process in Exercise 5 with gives $a = 5$, $b = 0.002$, and $c = 4$.

15. The San Diego Zoo discovered that because their flamingo population was too small, it would not reproduce until they borrowed some from Sea World. Scientists have discovered that certain gregarious animals require a minimum number of animals in a colony before they reproduce successfully. This is called the *Allee effect*. Consider the following model for the population of a gregarious bird species, where the population, N_n, is given in thousands of birds:

$$N_{n+1} = N_n + 0.2N_n\left(1 - \frac{1}{16}(N_n - 6)^2\right).$$

a. Assume that the initial population is $N_0 = 4$, then determine the population for the next two generations (N_1 and N_2).

b. Find all equilibria for this model.

c. The model above can be expanded to give

$$N_{n+1} = A(N_n) = \frac{3}{4}N_n + \frac{3}{20}N_n^2 - \frac{1}{80}N_n^3.$$

Find the derivative of $A(N)$. Evaluate the derivative $A'(N)$ at each of the equilibria found above and determine the local behavior of the solution near each of those equilibria.

d. Give a brief biological description of what your results imply about this gregarious species of bird.

16. The modeling of nerve cells often use a cubic response curve to the membrane potential V. Below we present a overly simple model for the membrane potential at discrete times for a nerve that can be quiescent or have repetitive spiking of action potentials. The simplified model is given by:

$$V_{n+1} = V_n + 0.07V_n(9 - (V_n - 4)^2).$$

a. Assume that the initial potential is $V_0 = 3$, then determine the membrane potential for the next three time intervals (V_1, V_2 and V_3).

b. Find all equilibria for this model.

c. The model above can be expanded to give

$$V_{n+1} = M(V_n) = 0.51V_n + 0.56V_n^2 - 0.07V_n^3.$$

Find the derivative of $M(V)$. Evaluate the derivative $M'(V)$ at each of the equilibria found above and determine the local behavior of the solution near each of those equilibria.

d. Give a brief biological description of what your results imply about the behavior of the nerve following different initial stimuli.