1. Consider the following linear discrete dynamical model:

$$y_{n+1} = 0.7y_n + 6.$$

Let $y_0 = 10$. Find y_1, y_2, and y_3. Also, find the equilibrium point, y_e. Does the solution approach the equilibrium (stable) or move away from the equilibrium (unstable)?

2. Consider the following linear discrete dynamical model:

$$z_{n+1} = 1.2z_n - 20.$$

Let $z_0 = 50$. Find z_1, z_2, and z_3. Also, find the equilibrium point, z_e. Does the solution approach the equilibrium (stable) or move away from the equilibrium (unstable)?

3. In the model for breathing, we could also have kept track of the Nitrogen (N_2) in the exhaled breath also. The mathematical model is the same as in the lecture notes,

$$c_{n+1} = (1 - q)c_n + q\gamma.$$

For the normal subject, we found that $q = 0.18$. The percent of N_2 in the atmosphere is 78%, so this gives $\gamma = 0.78$. Assume that the initial concentration of N_2 in the lungs is given by $c_0 = 0.7$. Find c_1, c_2, and c_3. Also, find the equilibrium point, c_e. Does the solution approach the equilibrium (stable) or move away from the equilibrium (unstable)?

4. Consider the model for breathing with Helium gas (He) as a tracer in the lungs. In the atmosphere, He occurs at 5.2 ppm. Suppose a subject with emphysema begins with a concentration of $c_0 = 100$ ppm. The mathematical model is the same as before,

$$c_{n+1} = (1 - q)c_n + q\gamma.$$

This subject has $q = 0.1$. Find c_1, c_2, and c_3. Also, find the equilibrium point, c_e. Does the solution approach the equilibrium (stable) or move away from the equilibrium (unstable)?

5. The lecture notes showed how the model could be used to determine the vital capacity of a subject. Suppose that the tidal volume, V_i, of the subject is 400 ml. For this experiment, Nitrogen, N_2, is used to determine the functional reserve capacity, V_r. (Note that $V_r = (1 - q)V_i/q$.) The mathematical model gives

$$c_{n+1} = (1 - q)c_n + q\gamma,$$

where $\gamma = 0.78$. You are given that $c_0 = 0.68$ and $c_1 = 0.694$. Use this information to find q, then determine the functional reserve capacity, V_r.

6. A woman with a chronic lung problem is found to have a vital capacity of only 1300 ml and a residual volume of 950 ml. Suppose that the tidal volume, V_i, of this patient is 350 ml. For this experiment, Helium, He, is used to determine the functional reserve capacity, V_r. (Recall that $V_r = (1 - q)V_i/q$.) The mathematical model gives

$$c_{n+1} = (1 - q)c_n + q\gamma,$$

where $\gamma = 5.2$ ppm.

a. The woman is given an enriched mixture of air to breathe that contains 30 ppm of He. Experimentally, it is found that her first 3 breaths after breathing the enriched mixture for a while have concentrations of He given by $c_0 = 30$, $c_1 = 25.8$ and $c_2 = 22.3$ ppm. Use c_0 and c_1 to find q, then determine the functional reserve capacity, V_r.

b. Use your model to find the expected concentration of Helium in this patient's 10^{th} breath, c_{10}. What is the equilibrium concentration of Helium in the patient's lungs?

c. If the functional reserve capacity is equal to the expiratory reserve volume plus the residual volume and the vital capacity is equal to the sum of the tidal volume and the inspiratory and expiratory reserve volumes, then use the data above to find the inspiratory and expiratory reserve volumes for this patient with chronic lung problems. Compare her values to those for a woman with normal lung function.

7. Consider a model with immigration given by

$$p_{n+1} = 1.05p_n + 200,$$

with an initial population of $p_0 = 1000$. Find the populations at the next three time intervals, p_1, p_2, and p_3.

8. The population in the U. S. at the turn of the last century is given in the following table (with population in millions).

Year	1900	1910	1920	1930
Population	76.0	92.0	105.7	122.8

a. Let $p_0 = 76.0$ and consider the Malthusian growth model

$$p_{n+1} = 1.17p_n,$$

where n is in decades. Find p_1, p_2, and p_3. Determine the percent error in these predictions compared to the actual values.

b. Again let $p_0 = 76.0$ and consider the Malthusian growth model with immigration. Assume that the immigration over a decade is approximately 3.0 million, then the model is given by

$$p_{n+1} = 1.14p_n + 3.0,$$

where n is in decades. Find p_1, p_2, and p_3. Determine the percent error in these predictions compared to the actual values. Notice that the actual growth rate is 3% lower in this model.

9. Below is data on several populations of herbivores in related areas.

p_0	p_1
70	90
100	150
150	250

The data is assumed to fit a discrete Malthusian model with emigration in the form

$$p_{n+1} = rp_n - \mu,$$

where $r - 1$ is the growth rate and μ is the emigration rate.

a. Use the data below to determine the updating function for this population, i.e., find r and μ and write the equation for this model.

b. Beginning with $p_0 = 100$, find the populations p_1, p_2, and p_3.

c. Find the equilibrium value and determine the stability of this equilibrium.

10. Below are data on the population of a species of moth that inhabits an island and breeds annually (then dies). If its offspring have a survival rate r, and there is a net (constant) influx of new moths from surrounding islands entering at a rate μ, then the population model has the form

$$P_{n+1} = rP_n + \mu.$$

a. From the data below determine the updating function for this population, *i.e.*, find r and μ. Then use this updating function to find the population of moths in 1993, 1994, and 1995.

b. Find all equilibria for this model. Based on your iterations in Part a, what is the stability of the equilibria? (If a solution moves closer to an equilibrium point, then it is probably stable. If it moves away, then it is most likely unstable.) What does this model predict will ultimately happen to the population of moths?

c. Graph the updating function along with the identity map, $P_{n+1} = P_n$. Determine all points of intersection.

Year	Moths
1990	6000
1991	5500
1992	5100