Sketch the curves of the functions below. List the relative maxima, relative minima, and points of inflection for each graph. Also, give the x and y-intercepts and any asymptotes if they exist. You should work these problems without the aid of a graphing calculator.

1. $y = 15 + 2x - x^2$,

2. $y = x^3 - 12x$,

3. $y = 2x^3 - 3x^2$,

4. $y = x^4 - 2x^2 + 1$,

5. $y = x^4 - 32x$,

6. $y = 2x + \dfrac{2}{x}$,

7. Body temperatures of animals undergo circadian rhythms. A subject's temperature is measured from 8 AM until midnight, and his body temperature, T (in °C), is best approximated by the cubic polynomial

$$T(t) = 0.002(t^3 - 45t^2 + 600t + 16000),$$

where t is in hours.

a. Find the rate of change in body temperature $\frac{dT}{dt}$. What is the rate of change in body temperature at noon $t = 12$?

b. Use the derivative to find when the maximum temperature of the subject occurs and when the minimum temperature of the subject occurs. What are the body temperatures at those times? State the intervals of time where the subject's body temperature is decreasing.

8. Over a 7 day period in the summer, data were collected on an algal bloom in the ocean. The population of algae (in thousand/cc), $P(t)$, were best fit by the cubic polynomial

$$P(t) = t^3 - 9t^2 + 15t + 30,$$

where t is in days.

a. Find the rate of change in population per day, $\frac{dP}{dt}$. What is the rate of change in the population on the first day, $t = 2$?

b. Use the derivative to find when the relative minimum and maximum populations of algae occur over the time of the survey. Give the populations at those times. Over what intervals of time is the population increasing?

c. Sketch a graph of this polynomial fit to the population of algae. Show clearly the maximum and minimum populations on your graph and include the populations at the beginning of the survey ($t = 0$) and at the end ($t = 7$).

9. In lab we saw the experimental fit of O_2 consumption (in μl/hr) after a blood meal by the beetle *Triatoma phyllosoma*. Below is a cubic polynomial fit to measurements for a different individual "kissing bug,"

$$Y(t) = \frac{1}{3}\,t^3 - 6\,t^2 + 20\,t + 120,$$

where t is in hours, for $0 \le t \le 12$.

a. Find the rate of change in O_2 consumption per hour, $\frac{dY}{dt}$. What is the rate of change in the O_2 consumption at $t = 6$?

b. Use the derivative to find when the minimum and maximum O_2 consumption for this beetle occurs during the experiment. Give the O_2 consumption at those times.

c. Sketch a graph of this polynomial fit to the O_2 consumption. Show clearly the maximum and minimum O_2 consumption on your graph and include the O_2 consumption at the beginning of the study ($t = 0$) and at the end ($t = 12$).

10. Many ecological studies require that the subject studied is correlated with the temperature of the environment (especially insects and plants). Over a 20 hour period, data are collected on the temperature, $T(t)$in degrees Celsius. The temperature data are found to best fit the cubic polynomial

$$T(t) = 0.01(1600 - 135t + 27t^2 - t^3),$$

where t is in hours (valid for $0 \le t \le 20$).

a. Find the rate of change in temperature per hour, $\frac{dT}{dt}$. What is the rate of change in the temperature at 3 AM, $t = 3$?

b. Use the derivative to find when the minimum and maximum temperatures occur. Give the temperatures at those times.

c. Sketch a graph of this polynomial fit to the temperature. Show clearly the maximum and minimum temperatures on your graph and include the temperatures at the beginning of the study ($t = 0$) and at the end ($t = 20$).

11. a. An impala is migrating across a field that has been fenced with a 180 cm fence. To escape it needs to jump this fence. Assume that the impala jumps the fence with just enough vertical velocity, v_0 to clear it. If the height (in cm) of the impala is given by

$$h(t) = v_0 t - 490t^2,$$

then find the velocity $v(t) = h'(t)$ of the impala at any time (in sec), $t \geq 0$, before hitting the ground.

b. Find when the velocity is equal to zero in terms of v_0. This is the time at the maximum height. Since the impala is 180 cm in the air at this time, use the equation for the height, $h(t)$ to compute the initial velocity, v_0, with which the impala must launch itself to clear the fence.

c. With the initial velocity computed above, determine how long the impala is in the air, when jumping over the fence.