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Monte Carlo Simulation of Operating-Room and Recovery-Room Usage

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The purpose of this paper is to provide an insight into the increased need for operating-room and recovery-room facilities and space, based on an increased bed complement. The problem is formulated into three primary questions: (1) How many more surgical procedures will be performed because of the increased bed capacity? (2) How much operating-room time and space will the surgical procedures require? (3) How much recovery-room time and space will the surgical procedures require? To answer these questions, a simulation model of the lengths of stay in the operating room and the recovery room is constructed by the Monte Carlo method, it is tested statistically and its results interpreted. This simulation model can facilitate planning, decision-making, and managerial control by providing management information.

WHEN A HOSPITAL decides to expand its bed complement, a decision must also be made as to the increased demands that will be made upon its ancillary departments—consideration of whether the present facilities are adequate for the increased bed complement often presents difficult and agonizing decisions.

The purpose of this paper is to provide hospitals with an insight into the increased need for operating-room and recovery-room facilities. The research was based on an increased bed complement of 144 medical-surgical beds at Deaconess Hospital. The problem is formulated into three primary questions: (1) How many more surgical procedures will Deaconess Hospital perform because of the increased bed capacity? (2) How much operating-room time and space will the surgical procedures require? (3) How much recovery-room time and space will the surgical procedures require?

To answer these questions, a simulation model of the average length of stay in the operating room and recovery room by the various types of patients is constructed by the Monte Carlo method; it is statistically tested, and its results interpreted. The simulation model can facilitate planning, decision making, and managerial control by providing management information. While the specific subject of interest in this paper is the increased use of the operating rooms and recovery rooms, the simulation method could be equally applicable to any other ancillary department. The simulation model also provides a means of conducting experiments in other hospitals.

DATA COLLECTION

DATA WERE COLLECTED from a sample of 445 of the patients at Deaconess Hospital in 1970, regarding the type of surgery performed—ophthalmology, gynecology, urology, orthopedic, ear-nose-throat (ENT), dental surgery, and other major surgery. Data on the average length of patient's stay in the hospital for the related type of surgery were also collected. The percentages of the various types of surgery and the average lengths of stay for the total population of patients in the hospital were inferred from the sample results.

In addition, data were collected from a sample of patients regarding the length of stay in the operating room by analyzing the charge tickets that were sent from the operating room to the business office, since the amount of operating-room time used by each patient is indicated on the charge tickets. The charge tickets for

TABLE I
LENGTH OF STAY IN THE OPERATING ROOM

Length of stay in hours	Frequency	Relative frequency
0.01–0.50	181	40.7
0.51–1.00	103	23.2
1.01–1.50	64	14.4
1.51–2.00	42	9.4
2.01–2.50	22	4.9
2.51–3.00	13	2.9
3.01–3.50	8	1.8
3.51–4.00	5	1.1
More than 4.00	7	1.6
Total	445	100.0

various minor-surgery procedures were tabulated and grouped into the time segment 0.01 to 0.50 hr. This course of action was taken because historical evidence indicates that these minor surgical procedures, for which there is a flat charge, average 0.47 of an hour in duration. (All time segments will be given in hundreths of an hour, rather than in minutes, because it simplifies the mathematical calculations. This practice will be the case throughout the paper.) The results of the collection of this data are summarized in Table I.

The frequencies of length of stay in the operating room have a Gamma distribution; more specifically, we have identified the distribution as a special kind of Gamma distribution. Chi-square tests support the hypothesis that the sample has a negative-exponential distribution.

PROJECTION OF THE INCREASE IN SURGICAL PROCEDURES

AN ANALYSIS OF THE *Monthly Service Report* of Deaconess Hospital shows that 42 per cent of medical-surgical (hereafter referred to as M/S) patients actually have surgery. Since 144 new M/S beds will be added to the present bed complement, it means that approximately 60 of the new beds will be utilized by patients who will have surgery.

To establish the absolute increase as well as the percentage increase in surgical cases, based on the increased bed count, a simple method of extrapolation was used for each of the types of surgery that are listed in the *Monthly Service Report.* The extrapolated figures are based on full bed utilization and the same patient mix between medical and surgical patients. Of the total number of surgical procedures performed (6,293) in 1970, 4.5 per cent were ophthalmology cases. The average length of stay for these cases was 7.4 days. Of the 60 beds referred to above as being utilized by surgical patients, 2.7 beds will be used by ophthalmology patients ($0.045 \times 60 = 2.7$). During a given year, 49 patients will be able to use each bed ($365/7.4 = 49$). Therefore, the estimated increase of ophthalmology cases per year will be 132 ($2.7 \times 49 = 132$). All other surgical services were analyzed and extrapolated in the same way.

TABLE II

INCREASE IN SURGICAL CASES BASED ON INCREASED BED COUNT

Type of surgery	Increase in number of surgical cases per year
Ophthalmology	132
Gynecology	282
Urology	264
Orthopedic	202
ENT	1,098
Dental surgery	715
Other major surgery	683
Total increase	3,376
Percentage increase	53.6%

The increase in surgical cases is illustrated in Table II. Adding the projected number of 3,376 to the total number of surgical procedures performed in 1970 when the old bed count was in effect, we arrive at a total of 9,669 projected surgical procedures when the new bed complement is fully utilized, or an increase of 53.6 per cent. This percentage increase will be applied to the daily surgical load to determine the projected daily load in the simulation.

GENERATION OF RANDOM NUMBERS

SINCE LENGTHS OF stay in the operating room are exponentially distributed, stays may be generated easily with the aid of random exponential numbers from a distribution having a mean of one. Each number drawn from such a distribution multiplied by the appropriate interarrival mean (1.03 hours) gives the simulated length of stay in the operating room.

The actual method of generating random exponential numbers (hereafter referred to as REN) is a simple conversion process from a uniform random numbers (hereafter referred to as URN). The conversion is accomplished by dividing the mid-point of the time interval by the mean amount that the patient spends in the operating room. [The exception to this approach is in the time segment 0.01 to 0.50. Since 95 per cent of the surgical cases in this time segment are in the discrete unit from 0.416 to 0.50, there is an upward biasing of the REN to reflect adequately the average length of stay in this time segment.] Table III shows the results.

Table III indicates that the URN is used for two different purposes. First, it serves as the basis upon which a conversion to a REN is made. Second, in the time interval from 0.01 to 0.50, the URN also serves as the basis upon which the decision is made as to whether the patient goes to the recovery room (RR). For example, a URN between 158 and 241 would be a urology case that goes to the recovery room, while a URN between 242 and 326 is a urology case that does not go to the recovery room.

TABLE III

CONVERSION OF URN TO REN

Type of surgery	Time interval	$P(t)^a$	URN	REN
ENT	0.01–0.50	15.8	000–157	0.490
Urology (to RR)		08.4	158–241	
Urology (no RR)		08.5	242–326	
Ophthalmology (no RR)		05.8	327–384	
All other surgery	0.51–1.00	23.6	385–620	0.728
" " "	1.01–1.50	14.6	621–766	1.214
" " "	1.51–2.00	09.0	767–856	1.699
" " "	2.01–2.50	05.5	857–911	2.184
" " "	2.51–3.00	03.4	912–945	2.700
" " "	3.01–3.50	02.1	946–966	3.155
" " "	3.51–4.00	01.3	967–979	3.641
" " "	More than 4.00	02.0	980–999	4.021

[a] These probabilities were calculated for the simulation model based on the cumulative exponential distribution, $P(t)=1-e^{-\mu t}$, where e=constant, μ=mean number of Poisson successes per length of stay (i.e., one divided by the mean length of stay in the operating room), and t=time; whereas the probabilities of Table I were based on the empirical data.

In the long run the mean amount of simulated times spent in the operating room should approximate the mean amount of observed times spent in the operating room. This hypothesis is supported by the test of significance (at the 0.01 level) for the difference between two means. The absolute difference between the two means (0.94 compared to 1.03) is attributed to the chance variation in the sample.

It has been substantiated by empirical evidence that the average length of stay in the recovery room for minor surgery patients is 1½ hr while the average length of stay in the recovery room for a major-surgery patient is 3 hr. [The data in Table III have been arranged so that the URN's from 000 to 384 represent minor surgery and the URN's from 385 to 999 represent major surgery, since empirical data indicate that approximately 38 per cent of the surgical cases are minor surgery and 62 per cent are major surgery.]

SIMULATION OF THE SYSTEM

Rules for the Simulation

For the simulation of the length of stay in the operating room and the recovery room, the following rules were applied. [All of the rules that are stated for this simulation have been substantiated by empirical evidence, where the need for substantiation is indicated. The numbers of operating rooms and recovery rooms assumed to be available were varied in the simulation, because these were the factors to be determined.]

1. Twenty-seven cases were simulated based on the increased bed complements.
2. The random numbers utilized to initiate the number generating part of the simulation must be different for each day.
3. All ENT, urology, and ophthalmology surgical cases have an average length of stay in the operating room of 0.50 hr.
4. Fifty per cent of the urology surgical cases do not go to the recovery room because they were performed under a local anesthetic. (Whether or not the urology case goes to the recovery room is governed by the URN in Table III.)
5. All ENT surgical cases go to the recovery room. Since these are minor surgical cases, their length of stay is 1½ hr.
6. None of the ophthalmology cases go to the recovery room. The few ophthalmology cases that actually do go to the recovery room are balanced out by the few ENT cases that do not go to the recovery room.
7. The starting time for the beginning of the surgical schedule is 7.50 AM.
8. The necessary 'make-ready' time from the time that one surgical case leaves the operating room until it is ready to receive the next is 0.25 hr.
9. It will take 0.08 hr to transport the patient from the operating room to the recovery room.
10. The necessary 'make ready' time from the time that the patient leaves the recovery room until his bed is ready for the next occupant is 0.25 hr.
11. The first operating room to be vacated is the first one to be put back into use when the need arises.
12. The first recovery-room bed to be vacated is the first one to be put back into use when the need arises.

Explanation of the Simulation Model

Table IV shows an example portion of the simulation; the following explanation is keyed to this table.

1. A URN was selected from a table of random numbers.
2. The URN was converted to a REN by use of Table III. Since the URN was 862, the converted value of the REN was 2.18.
3. The REN was multiplied by the mean amount of time spent in the operating room (1.03 hr from Table I). [*See* Note 1.] The result shows that the simulated length of the operation was 2.25 hr.

TABLE IV

AN EXAMPLE OF THE SIMULATION

URN	REN	Length of operation	Time operation begins	Time operation ends	Operating room number	Recovery room Yes	Recovery room No	Time recovery begins	Time recovery ends	RR bed no.	Time RR bed available
862	2.18	2.25	7.50	9.75	1	X		9.83	12.83	6	13.08
872	2.18	2.25	7.50	9.75	2	X		9.83	12.83	7	13.08
472	0.73	0.75	7.50	8.25	3	X		8.33	11.33	1	11.58
772	1.70	1.75	7.50	9.25	4	X		9.33	12.33	5	12.58
436	0.73	0.75	7.50	8.25	5	X		8.33	11.33	2	11.58
142	0.49	0.50	8.50	9.00	3	X		9.08	10.58	3	10.83
200	0.49	0.50	8.50	9.00	5	X		9.08	10.58	4	10.83
000	0.49	0.50	9.25	9.75	3	X		9.83	10.58	8	10.83
749	1.21	1.25	9.25	10.50	5	X		10.58	13.58	9	13.83
992	4.02	4.15	9.50	13.65	4	X		13.73	16.73	8	16.98
557	0.73	0.75	10.00	10.75	1	X		10.83	13.83	3	14.08
528	0.73	0.75	10.00	10.75	2	X		10.83	13.83	4	14.08
916	2.70	2.78	10.00	12.18	3	X		12.86	15.86	5	16.11
230	0.49	0.50	10.75	11.25	5	X		11.33	12.83	8	13.08
681	1.21	1.25	11.00	12.25	1	X		12.33	15.33	10	15.58
435	0.73	0.75	11.00	11.75	2	X		11.83	14.83	1	15.08
396	0.73	0.75	11.50	12.25	5	X		12.33	15.33	2	15.58
655	1.21	1.25	12.00	13.25	2	X		13.33	16.33	6	16.58
398	0.73	0.75	12.50	13.25	1	X		13.33	16.33	7	16.58
934	2.70	2.78	12.50	15.28	5	X		15.36	18.36	1	18.61
683	1.21	1.25	13.03	14.28	3	X		14.36	17.36	3	17.61
913	2.70	2.78	13.50	16.28	2	X		16.36	19.36	10	19.61
401	0.73	0.75	13.50	14.25	1	X		14.33	17.33	9	17.58
569	0.73	0.75	13.90	14.65	4	X		14.73	17.73	4	17.98
889	2.18	2.25	14.50	16.75	1	X		16.83	19.83	2	20.08
619	0.73	0.75	16.53	17.28	3	X		17.36	20.36	5	20.61
432	0.73	0.75	14.90	15.65	4	X		15.73	18.73	11	18.98
889	2.18	2.25	7.50	9.75	1	X		9.83	12.83	7	13.08
396	0.73	0.75	7.50	8.25	2	X		8.33	11.33	1	11.58
358	0.49	0.50	7.50	8.00	3		X	—	—	—	—
715	1.21	1.25	7.50	8.75	4	X		8.83	11.83	3	12.08
502	0.73	0.75	7.50	8.25	5	X		8.33	11.33	2	11.58
068	0.49	0.50	8.25	8.75	3	X		8.83	10.33	4	10.58
604	0.73	0.75	8.50	9.25	2	X		9.33	12.33	5	12.58
270	0.49	0.50	8.50	9.00	5		X	—	—	—	—
228	0.49	0.50	9.00	9.50	4	X		9.58	11.08	6	11.33
782	1.70	1.75	9.00	10.75	3	X		10.83	13.83	4	14.08
379	0.49	0.50	9.25	9.75	5		X	—	—	—	—
093	0.49	0.50	9.50	10.00	2	X		10.08	11.58	8	11.83
011	0.49	0.50	9.75	10.25	4	X		10.33	11.83	9	12.08
648	1.21	1.25	10.00	11.25	1	X		11.33	14.33	6	14.58
528	0.73	0.75	10.00	10.75	5	X		10.83	13.83	10	14.08
987	4.02	4.15	10.25	14.40	2	X		14.48	17.48	5	17.73
214	0.49	0.50	10.50	11.00	4	X		11.08	12.58	11	12.83
474	0.73	0.75	11.00	11.75	3	X		11.83	14.83	2	15.08
238	0.49	0.50	11.00	11.50	5	X		11.58	13.08	1	13.33
045	0.49	0.50	11.25	11.75	4	X		11.83	13.33	5	13.58
408	0.73	0.75	11.50	12.25	1	X		12.33	15.33	3	15.58
116	0.49	0.50	11.75	12.25	5	X		12.33	13.83	9	14.08
209	0.49	0.50	12.00	12.50	3	X		12.58	14.08	5	14.33
048	0.49	0.50	12.00	12.50	4	X		12.58	14.08	11	14.33
393	0.73	0.75	12.50	13.25	1	X		13.33	16.33	7	16.58

TABLE IV—*Continued*

URN	REN	Length of operation	Time operation begins	Time operation ends	Operating room number	Recovery room		Time recovery begins	Time recovery ends	RR bed no.	Time RR bed available
						Yes	No				
550	0.73	0.75	12.50	13.25	5	X		13.33	16.33	1	16.58
306	0.49	0.75	12.75	13.50	3		X	—	—	—	—
979	3.64	3.75	7.50	11.25	1	X		11.33	14.33	10	14.58
375	0.49	0.50	7.50	8.00	2		X	—	—	—	—
075	0.49	0.50	7.50	8.00	3	X		8.08	9.58	1	9.83
649	1.21	1.25	7.50	8.75	4	X		8.83	11.83	3	12.08
235	0.49	0.50	7.50	8.00	5	X		8.08	9.58	2	9.83
623	1.21	1.25	8.25	9.50	2	X		9.58	12.58	5	12.83
202	0.49	0.50	8.25	8.75	3	X		8.83	10.33	4	10.58
828	1.70	1.75	8.25	10.00	5	X		10.08	13.08	6	13.33
428	0.73	0.75	9.00	9.75	4	X		9.83	12.83	1	13.08
617	0.73	0.75	9.00	9.75	3	X		9.83	12.83	2	13.08
730	1.21	1.25	9.75	11.00	2	X		11.08	14.08	9	14.33
467	0.73	0.75	10.00	10.75	4	X		10.83	13.83	7	14.08
077	0.49	0.50	10.00	10.50	3	X		10.58	12.08	4	12.33
202	0.49	0.50	10.25	10.75	5	X		10.83	12.33	8	12.58
809	1.70	1.75	10.75	12.50	3	X		12.58	15.58	4	15.83
987	4.02	4.15	11.00	15.15	4	X		15.23	18.23	9	18.48
032	0.49	0.50	11.00	11.50	5	X		11.58	13.08	11	13.33
327	0.49	0.50	11.25	11.75	2		X	—	—	—	—
607	0.73	0.75	11.50	12.25	1	X		12.33	15.33	3	15.58
258	0.49	0.50	11.75	12.25	5		X	—	—	—	—
923	2.70	2.78	12.00	14.78	2	X		14.86	17.86	6	18.11
724	1.21	1.25	12.75	14.00	3	X		14.08	17.08	5	17.33
830	1.70	1.75	12.50	14.25	1	X		14.33	17.33	1	17.58
529	0.73	0.75	12.50	13.25	5	X		13.33	16.33	8	16.58
773	1.70	1.75	13.50	15.25	5	X		15.33	18.33	11	18.58
107	0.49	0.50	14.25	14.75	3	X		14.83	16.33	2	16.58
617	0.73	0.75	14.50	15.25	1	X		15.33	18.33	7	18.58
674	1.21	1.25	7.50	8.75	1			8.83	11.83	3	12.08
107	0.49	0.50	7.50	8.00	2	X		8.08	9.58	1	9.83
341	0.49	0.50	7.50	8.00	3		X	8.08	—	—	—
465	0.73	0.75	7.50	8.25	4			8.33	11.33	2	11.58
247	0.49	0.50	7.50	8.00	5		X	8.08	—	—	—
115	0.49	0.50	8.25	8.75	2	X		8.83	10.33	4	10.58
215	0.49	0.50	8.25	8.75	3	X		8.83	10.33	5	10.58
017	0.49	0.50	8.25	8.75	5	X		8.83	10.33	6	10.58
394	0.73	0.75	8.50	9.25	4	X		9.33	12.33	7	12.58
527	0.73	0.75	9.00	9.75	1	X		9.83	12.83	1	13.08
093	0.49	0.50	9.00	9.50	2	X		9.58	11.08	8	11.33
160	0.49	0.50	9.00	9.50	3	X		9.58	11.08	9	11.33
912	2.70	2.78	9.00	11.78	5	X		11.86	14.86	9	15.11
008	0.49	0.50	9.50	10.00	4	X		10.08	11.58	10	11.83
409	0.73	0.75	9.75	10.50	2	X		10.58	13.58	4	13.83
747	1.21	1.25	9.75	11.00	3	X		11.08	14.08	6	14.33
113	0.49	0.50	10.00	10.50	1	X		10.58	12.08	5	12.33
537	0.73	0.75	10.25	11.00	4	X		11.08	14.08	11	14.33
945	2.70	2.78	10.75	13.53	2	X		13.61	16.61	5	16.86
868	2.18	2.25	10.75	13.00	1	X		13.08	16.08	2	16.33
785	1.70	1.75	11.25	13.00	3	X		13.08	16.08	10	16.33
226	0.49	0.50	11.25	11.75	4	X		11.83	13.33	8	13.58
823	1.70	1.75	12.00	13.75	4	X		13.83	16.83	7	17.08
706	1.21	1.25	12.03	13.28	5	X		13.36	16.36	3	16.61
765	1.21	1.25	13.25	14.50	1	X		14.58	17.58	1	17.83
096	0.49	0.50	13.25	13.75	3	X		13.83	15.33	8	15.58
744	1.21	1.25	13.53	14.78	5	X		14.86	18.86	12	19.11

4. Adding the simulated length of the operation to the starting time of the operation (7.50 hr), we find that the ending time of the operation was 9.75 A.M.

5. Adding 0.08 hr to the ending time of the operation, we find that the patient arrived at the recovery room at 9.83 A.M.

6. In reviewing the various times that patients arrived at the recovery room, we find that this first patient of the day on the surgical schedule was actually the sixth patient to arrive in the recovery room. Looking at the column headed "time recovery room available," we find that the first bed for reuse will not become available until 10.83 A.M. Therefore, we must assign the patient to a new bed—in this case bed number 6.

7. Since this is a major operation (it will be remembered that all surgical cases with a URN of 385 or greater will be considered as major surgeries), 3 hr are added to the arrival time to calculate that departure time, which is 12.83 P.M.

8. Adding 0.25 hr of make-ready time to the time of departure, we find that bed number 6 will be available for reuse at 13.08 P.M.

CONCLUSIONS

THE SIMULATIONS presented in Table IV utilized five operating rooms. Furthermore, the surgical schedule called for 27 patients on each of the four days that were simulated for purposes of illustration. It is apparent from these simulation runs that it is a relatively simple matter to vary the number of surgical patients for the day as well as vary the number of operating rooms used in the simulation.

On the basis of using five operating rooms and having a surgical schedule of 27 patients, the maximum number of recovery room beds in use at any one time was 12 while the minumum number in use was 11. [*See* Note 2.]

Furthermore, this simulation should give an insight into the optimum length of time to have the recovery room open if it is not manned 24 hr a day. In this simulation the latest departure time from the recovery rooms was 20.61 P.M.

In addition, this simulation model should give an insight into the optimum number of operating rooms to use, based on the number of patients on the surgical schedule. For example, if the number of patients on the surgical schedule had been 15 instead of 27, the use of five operating rooms would have completed the surgical schedule before noon. This would be an obviously inefficient use of the operating-room suite and personnel.

This method of simulation is extremely flexible. It allows the various levels of sophistication. For example, one could use 5-min time segments instead of the 30-min time segments chosen for this illustration. Further, the recovery room usage could be simulated in the same way that the operating-room usage was simulated instead of adding a raw mean to indicate length of stay in the recovery room. This would give a much higher level of sophistication if the need were indicated.

In general, this method gives a close approximation to reality under conditions when it is not possible to ascertain by observation the operation of a department. It was found to be extremely accurate when it did become possible to observe the operation of the department. The uses of the method are limited only by the imagination and ingenuity of the user.

NOTES

1. The reason for the apparent redundant effort (i.e., dividing the midpoint of the time interval that the patient spends in the operating room by mean time spent in the operating room in order to arrive at the REN and then multiplying that same REN by the mean patient stay to determine the length of stay in the operating room), is that the model was simplified to use a single average REN for each time segment. If individual random exponential numbers had been generated to determine the length of stay in the operating room, it becomes readily apparent why the random exponential number is multiplied by the mean length of stay in the operating room in order to determine the simulated length of stay in the operating room. It was found that the latter approach did not add significantly to the accuracy of the model, so the simplified method was adopted.

2. The simulation was conducted to determine the number of operating-room and recovery-room beds to be constructed in a building program that also included the addition of 144 medical-surgical beds. The simulation was conducted using 3, 4, 5, and 6 operating rooms. Based on 27 surgical cases per day, which was the predicted new surgical load due to the increased bed complement, the optimum number of operating rooms was found to be 5, and there would consistently be a need for at least 12 recovery-room beds.

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