

**1. Optimization:**

- a. A rectangular study plot is bounded on one side by a river, and the other three sides are to be blocked off by a fence. Find the dimensions of the plot that maximizes the area enclosed with 200 meters of fence.
- b. Two identical rectangular study plots are created with each region enclosing  $10 \text{ m}^2$  of field. The two study plots are side-by-side, sharing a common boundary. Find the dimensions of each plot area that minimizes the amount of fence needed.
- c. An open box with its base having a length four times its width is to be constructed with  $1200 \text{ cm}^2$  of material. Find the dimensions that maximize the volume of this box.
- d. A closed rectangular box must have  $5000 \text{ cm}^3$  of space, and its base must have its length be twice its width. Find the dimensions of this box, which is constructed with the least amount of building material possible.
- e. A right circular cylindrical container with only a bottom must have a volume of  $4500 \text{ cm}^3$ . Find the dimensions so that it is constructed with the least amount of material possible.

**2. Trigonometric definitions:** Use the definitions of sine and cosine to give your answer for the other function using only fractions with integers and square roots. The angle measurement uses your scientific calculator.

- a. Let  $\sin(\theta) = \frac{3}{4}$ , then find  $\cos(\theta)$  (not in decimal form). Also, determine the value of  $\theta$  in radians (using your scientific calculator).
- b. Let  $\cos(\theta) = \frac{2}{7}$ , then find  $\sin(\theta)$  (not in decimal form). Also, determine the value of  $\theta$  in radians (using your scientific calculator).

3. A damped spring-mass system has a solution of the form

$$y(t) = 12 + 7e^{-0.3t} \sin(5t),$$

where  $y(t)$  measures the distance in centimeters from the equilibrium position and  $t$  is in seconds.

- a. Determine the times when  $y(t) = 12$ .
- b. Find the velocity of the mass by computing the derivative,  $v(t) = y'(t)$ .
- c. Find the times  $t \geq 0$ , when the mass is at its absolute maximum and absolute minimum. Also, give the maximum and minimum displacements at those times. Sketch a graph for the position of this mass for  $t \in [0, \pi]$ .