1. **(1 pt) mathbioLibrary/setABiocLabs/Lab121_L2_yeast.pg**

Because of the accuracy of WebWork, you should use 4 or 5 significant figures on this problem.

We have studied the discrete logistic growth model and seen the difficulties computing and analyzing populations using this model. Most biologists use the continuous version of the logistic growth model for their studies of populations. This model is used very extensively and can be written with the following formula

$$P(t) = \frac{P_0 M}{P_0 + (M - P_0)e^{-rt}},$$

where P_0 is the initial population, M is the carrying capacity of the population, and r is the Malthusian growth rate (early exponential growth rate) of the culture. Below is a table with data from Gause [1] on a growing culture of the yeast, *Schizosaccharomyces kephir* (a contaminant culture of brewer's yeast).

t (hr)	0	15	36.5	75	99	122
$P(t)$ (vol)	1.1	2.29	4.48	5.71	5.75	5.83

a. Use Excel to find the best values of parameters P_0, M, and r. Include the sum of squares error. Also, write the complete formula with the best parameters fit to the model.

$P_0 =$ _______

$M =$ _______

$r =$ _______

$SSE =$ _______

$P(t) =$ __________________________

Give the model population prediction at times 15 and 75 and find the percent error at each of these times from the actual data given:

Population at $t = 15$ _______

Percent Error at $t = 15$ _______

Population at $t = 75$ _______

Percent Error at $t = 75$ _______

b. The Malthusian growth model satisfies the equation:

$P_E(t) = P_0 e^{rt}$,

where we take the parameters, P_0 and r, found in Part a. If the value of r gives the Malthusian growth rate for low populations, then use this to determine the doubling time for this culture of yeast.

Malthusian Doubling Time = _______

Use the logistic growth model to find the time for this culture to double from its initial population (using the best fitting P_0). Determine the percent error between the Malthusian growth doubling time and the logistic growth doubling time (assuming that the logistic growth doubling time is the more accurate).

Logistic Doubling Time = _______

Percent Error = _______

c. In your Lab report, create a graph with the data and the best fitting logistic growth model, $P(t)$, for $t \in [0, 150]$. Add a graph showing the Malthusian growth model, $P_E(t)$, for $t \in [0, 30]$. As noted above, M is the carrying capacity of the population. Write a brief biological interpretation of this parameter and describe what your value of M says about what happens to this experimental culture of yeast. Include a discussion of how well the two models simulate the data.

d. The growth rate for a culture can be found by taking the derivative of the population function. Differentiate the logistic growth function $P(t)$ with the parameters found in Part a.

$P'(t) =$ __________________________

Recall that geometrically the derivative at a point is the tangent line to the curve at the point, which also can be viewed as the limit of the secant lines connecting the point to other points on the curve as those points approach the given point. When we have data, we can approximate the derivative at the midpoint between two data points by finding the slope of the secant line connecting the two data points.

Let t_m be the midpoint between the data at 36.5 and 75. Approximate the derivative at t_m by finding the slope of the of the line connecting the data at 36.5 and 75.

$t_m =$ _______

$P'(t_m) \approx$ _______

Also, find the exact value of the derivative by using the best fitting logistic growth model above and computing $P'(t_m)$.

$P'(t_m) =$ _______

Find the percent error between the approximate error found from slope of the line connecting the data and the derivative at t_m.

Percent Error in derivative = _______

e. In your Lab report, create a graph of the derivative of the logistic growth function, $P'(t)$. Also, compute the slope of the secant lines connecting each successive pair of data points and assign these values to the midpoint between the time data points, so if two successive data points are (t_i, P_i) and (t_{i+1}, P_{i+1}) with midpoint

$t_m = \frac{t_i + t_{i+1}}{2}$

and slope

$s_m = \frac{P_{i+1} - P_i}{t_{i+1} - t_i}$,

then add to your graph of the derivative the data approximations to the derivative (as points on your graph) with the coordinates (t_m, s_m). Write a brief paragraph that discusses the shape of the curve created by the derivative. What does this curve say about the growth of the population of this yeast? Discuss how well you can use the data to approximate the derivative. Can you use these approximates to the derivative to determine where the growth rate is at a maximum?

f. The turning point of the population or the mid-log phase for this culture of yeast is where the growth of the culture is at a maximum. (This is also the point of inflection for the original logistic growth function, $P(t)$. Find when the logistic growth function reaches the turning point by finding the maximum of the derivative of the logistic growth function, $P'(t)$. Write the time of the turning point, the maximum growth that you find, and the population (volume) of the culture at this time.

$t_{max} =$ ________

$P(t_{max}) =$ ________

$P'(t_{max}) =$ ________

g. Write a brief paragraph that discusses how you found the turning point of the yeast population. Without graphing the function, $P''(t)$, describe some of the properties that this function should have, such as where it is positive or negative and where it has any t-intercepts. Summarize your modeling efforts in this lab and briefly discuss the strengths and weaknesses of using the logistic growth model.

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[1] G. F. Gause, Struggle for Existence, Hafner, New York, 1934.