1. (1 pt) mathbioLibrary/setABiocLabs/Lab121_K6_radio_moly.pg

Because of the accuracy of WebWork, you should use 5 or 6 significant figures on this problem.

Radioactive isotopes are very important in a number of medical applications. Most of these isotopes are created in nuclear reactors, then processed and delivered to hospitals for various applications. These isotopes are often very short-lived and must be shipped weekly for medical procedures. Molybdenum-99 (Mo-99) is an isotope that results from the fission of U-235 in nuclear reactors (and atomic bombs). It is relatively short-lived, and it decays into Technetium-99, which has a very short half-life. The Technetium-99 (Tc-99) produces very high-quality medical images for cancers, heart disease, and other serious problems. Unfortunately, many of the nuclear reactors licensed to produce Mo-99 (none in the U.S.) are aging and often need to shut down for repairs. Recently, this resulted in a 70 percent drop in world supplies [1], which meant that significant medical procedures had to be delayed or cancelled. In this problem, we use the nuclear chemistry of Mo-99 and Tc-99 to determine half-lifes and show how to calculate supplies.

a. Below is a table showing the amount of Mo-99 and Tc-99 at various times.

t (hr)	$M(t)$	$W(t)$
0	19.64	0
5	18.55	74.7
10	17.59	109
20	15.82	134.8
40	12.89	124.4
60	10.43	103
80	8.42	81.1

Mo-99 decays with a characteristic half-life according to the equation:

$$M(t) = M_0 e^{-r_1 t},$$

where t is in hours. Use the data on Mo-99 and Excel's Trendline with an exponential fit to find the best parameters, M_0 and r_1. Also, compute the sum of square errors between the model and the data.

$M_0 =$ ______

$r_1 =$ ______

$M(t) =$ ______________

$SSE =$ ______

Use this information to determine the half-life of Mo-99.

Half-Life = ______ hrs

Find the M-intercept and the horizontal asymptote for this model.

M-intercept = ______

Horizontal Asymptote $M =$ ______

The rate of radioactive decay, which determines the amount of radioactivity from Mo-99, is given by the derivative of $M(t)$. Find this derivative.

$M'(t) =$ ______________

b. In your Lab report, create a graph with the data and the model for the radioactive decay of Mo-99 for $t \in [0, 100]$. Create a short paragraph that briefly describes how well the model simulates the data.

c. Molybdenum-99 undergoes a gamma decay to produce the radioactive element Technetium-99, which is the primary isotope that is used for the medical images. The equation that describes the amount of Tc-99 coming from the Mo-99 is given by

$$W(t) = W_0(e^{-r_1 t} - e^{-r_2 t}),$$

where r_1 comes from your calculation above. Find the least squares best fit of this model to the data in the Table above, using Excel's Solver to find the best parameters, W_0 and r_2. (For starting values, take $W_0 = 200$ and $r_2 = 0.1$.) Also, compute the sum of square errors between the model and the data.

$W_0 =$ ______

$r_2 =$ ______

$W(t) =$ ______________

$SSE =$ ______

The decay constant r_2 gives the decay rate of Tc-99. Use this information to determine the half-life of Tc-99.

Half-Life = ______ hrs

Find the W-intercept and the horizontal asymptote for this model.

W-intercept = ______

Horizontal Asymptote $W =$ ______

The rate of radioactive decay, which determines the amount of radioactivity from Tc-99, is given by the derivative of $W(t)$. Find this derivative.

$W'(t) =$ ______________

Find the time, t_{max}, where the maximum amount of Tc-99 occurs and the value of $W(t_{max})$.

$t_{max} =$ ______ hrs

$W(t_{max}) =$ ______ g.

After the maximum occurs, the amount of Tc-99 first decreases more and more rapidly, then after a point of inflection this rate of radioactive decay decreases. Find the second derivative of $W(t)$.

$W''(t) =$ ____________________

Find the point of inflection for $W(t)$.

Time for point of inflection $t_i =$ ______hrs.

Determine both the amount of Tc-99 at that time and the rate of radioactive decay at that time, i.e., find

$W(t_i) =$ ______ g.

$W'(t_i) =$ ______ g/hr.

d. In your Lab report, create a graph with the data and the model for the radioactive decay of Tc-99 for $t \in [0, 100]$. Create a short paragraph that briefly describes how well the model simulates the data. Use the model above to help explain why hospitals need to have weekly shipments for their medical imaging.

[1] Science News, ”Desparately seeking Moly,” by Janet Raloff, Sept. 26, 2009, p. 16-20.