1. **(1 pt) mathbioLibrary/setABiocLabs/Lab121_I4_fish_cm.pg**
Because of the accuracy of WebWork, you should use 5 or 6 significant figures on this problem.

The growth of fish has been shown to satisfy a model given by the von Bertalanffy equation:

$$L(t) = L_\infty(1 - e^{-bt}),$$

where L_∞ and b are constants that fit the data.

a. Below are growth data for the Striped Marlin (*Tetrapturus audax*) [1].

Age (yr)	Length (cm)	Age (yr)	Length (cm)
1	89	6	183
2	132	7	186
3	159	8	187
4	172	9	188
5	179	10	189

Find the least squares best fit of the data to the von Bertalanffy equation above. Give the values of the constants L_∞ and b and write the model with these constants. Include the value of the least sum of squares error fitting the data.
$L_\infty =$ _____ cm
$b =$ _____
$L(t) =$ _____ cm
$SSE =$ _____
Find the L-intercept and the horizontal asymptote for the length of the Striped Marlin.
L-intercept = _____ cm
Horizontal Asymptote $L =$ _____ cm
Give the model prediction at age 6 and 10 and find the percent error at each of these ages from the actual data given (assuming the actual data is the more accurate value):
Length at age 6 = _____ cm
Percent Error at 6 = _____
Length at age 10 = _____ cm
Percent Error at 10 = _____

b. In your Lab report, create a graph with the data and the von Bertalanffy model for $t \in [0, 15]$. Create a short paragraph that briefly describes how well the model simulates the data and what the maximum size of this fish can be.

c. For this part of the problem, we use the data to compute average growth rates for Striped Marlin. The average growth rate is found by the following formula:

$$g_a(t_m) = \frac{L(t_2) - L(t_1)}{t_2 - t_1} \quad \text{where} \quad t_m = \frac{t_1 + t_2}{2},$$

where $L(t_1)$ and $L(t_2)$ are two successive data measurements of length and t_m is the midpoint between the ages that the fish was measured. We determine all the average growth rates from successive measurements from the Table above.
For $t_1 = 1$ and $t_2 = 2$, then $g_a(1.5) =$ _____ cm/yr.
For $t_1 = 2$ and $t_2 = 3$, then $g_a(2.5) =$ _____ cm/yr.
For $t_1 = 3$ and $t_2 = 4$, then $g_a(3.5) =$ _____ cm/yr.
For $t_1 = 4$ and $t_2 = 5$, then $g_a(4.5) =$ _____ cm/yr.
For $t_1 = 5$ and $t_2 = 6$, then $g_a(5.5) =$ _____ cm/yr.
For $t_1 = 6$ and $t_2 = 7$, then $g_a(6.5) =$ _____ cm/yr.
For $t_1 = 7$ and $t_2 = 8$, then $g_a(7.5) =$ _____ cm/yr.
For $t_1 = 8$ and $t_2 = 9$, then $g_a(8.5) =$ _____ cm/yr.
For $t_1 = 9$ and $t_2 = 10$, then $g_a(9.5) =$ _____ cm/yr.

We have noted that growth rates are derivatives. Use Maple to find the derivative of the von Bertalanffy model, $L'(t)$, found in Part a.
$L'(t) =$ _____ cm/yr.
Use this formula to find the growth rate at the times listed below, then determine the percent error using the model growth rate from the actual growth rate determined from the data (the better value).
At age $t = 2.5$, $L'(2.5) =$ _____ cm/yr.
Percent error = _____
At age $t = 4.5$, $L'(4.5) =$ _____ cm/yr.
Percent error = _____
At age $t = 6.5$, $L'(6.5) =$ _____ cm/yr.
Percent error = _____
At age $t = 8.5$, $L'(8.5) =$ _____ cm/yr.
Percent error = _____

d. In your Lab Report, graph as data points the average growth rates that you computed above. To this graph add the model growth rate computed from the derivative of the von Bertalanffy equation for $t \in [0, 12]$. Briefly discuss how well the derivative of the model simulates the actual measured growth rates.

2

[1] M. G. Hinton. Status of Blue Marlin in the Pacific Ocean. Website accessed 1/04.
[2] J. H. Uchiyama and T. K. Kazama. Updated Weight-on-length relationaships for pelagic fishes caught in the central north Pacific Ocean and bottomfishes from the Northwestern Hawaiian Islands, www.nmfs.hawaii.edu/adminrpts/PIFSC Admin Rep 03-01.pdf, (accessed 1/04)