

**1. (1 pt) mathbioLibrary/setABioc2Labs/Lab122\_J3\_predator-prey.pg**

Because of the accuracy of WebWork, you should use 5 or 6 significant figures on this problem.

G. F. Gause in his book *The Struggle for Existence* studied a number of competition and predator-prey systems in the lab. One predator-prey system that he studied was the interaction between the predator *Didinium nasutum* and its prey *Paramecium caudatum*. Below is a table based on one of his experiments.

Day	<i>P. caud.</i>	<i>D. nasu.</i>	Day	<i>P. caud.</i>	<i>D. nasu.</i>
0	2	5	9	4	5
1	4	2	10	3	3
2	9	2	11	12	1
3	21	5	12	22	3
4	32	8	13	42	14
5	51	19	14	28	30
6	26	28	15	15	27
7	12	17	16	5	14
8	6	12	17	2	5

a. The classic predator-prey model originally described by Lotka and Volterra in the 1920s is given by:

$$\frac{dP}{dt} = a_1P - a_2PN$$

$$\frac{dN}{dt} = -b_1N + b_2PN$$

where  $P$  is the population of *Paramecium caudatum* and  $N$  is the population of *Didinium nasutum*. (It should be noted that though this model is widely studied, it has mathematical properties that make it a weak model.) There is not an exact solution to this differential equation, so you need to use an Euler's method to approximate the solution for this system of differential equations. The Euler's formula for this system is given by

$$P_{n+1} = P_n + h(a_1P_n - a_2P_nN_n)$$

$$N_{n+1} = N_n + h(-b_1N_n + b_2P_nN_n)$$

Notice that this is simply a discrete dynamical system, so you can simulate this system very much like other discrete dynamical systems.

The data oscillates, which creates challenges to finding the best fitting parameters. To facilitate your finding the best fitting parameters, you are given the following set of initial guesses:  $P(0) = 5$ ,  $N(0) = 10$ ,  $a_1 = 0.8$ ,  $a_2 = 0.06$ ,  $b_1 = 0.8$ , and  $b_2 = 0.06$ . Not only is this model structurally unstable and sensitive to choice of parameters, but Euler's method poorly tracks the solution unless the stepsize  $h$  is small. Use  $h = 0.1$  and simulate this model for  $t \in [0, 17]$ .

Compute the sum of square errors at the times, where data is given, using the formula:

$$\sum_{i=0}^{17} ((P_{n_i} - P_d(t_i))^2 + (N_{n_i} - N_d(t_i))^2),$$

where  $P_{n_i}$  and  $N_{n_i}$  are the Euler simulations at the appropriate times  $t_i$  with the data evaluated at those times,  $P_d(t_i)$  and  $N_d(t_i)$ . Use Excel's Solver to find the least sum of square errors, varying the parameters  $P(0)$ ,  $N(0)$ ,  $a_1$ ,  $a_2$ ,  $b_1$ , and  $b_2$ . The best fitting parameters are:

$$P(0) = \underline{\hspace{2cm}} \text{ and } N(0) = \underline{\hspace{2cm}}$$

$$a_1 = \underline{\hspace{2cm}} \text{ and } a_2 = \underline{\hspace{2cm}}$$

$$b_1 = \underline{\hspace{2cm}} \text{ and } b_2 = \underline{\hspace{2cm}}$$

$$\text{Sum of Square Errors} = \underline{\hspace{2cm}}$$

From the simulation, estimate the number of *Paramecium caudatum*,  $P(t_i)$ , and *Didinium nasutum*,  $N(t_i)$ , when  $t = 2, 10$ , and 12 days. Also, Compute the percent error with the actual data (assuming the data give the best values)

$$\text{At } t = 2 \text{ days, } P(2) \approx \underline{\hspace{2cm}} \text{ with Percent Error} = \underline{\hspace{2cm}}$$

$$\text{and } N(2) \approx \underline{\hspace{2cm}} \text{ with Percent Error} = \underline{\hspace{2cm}}$$

$$\text{At } t = 10 \text{ days, } P(10) \approx \underline{\hspace{2cm}} \text{ with Percent Error} = \underline{\hspace{2cm}}$$

$$\text{and } N(10) \approx \underline{\hspace{2cm}} \text{ with Percent Error} = \underline{\hspace{2cm}}$$

$$\text{At } t = 12 \text{ days, } P(12) \approx \underline{\hspace{2cm}} \text{ with Percent Error} = \underline{\hspace{2cm}}$$

$$\text{and } N(12) \approx \underline{\hspace{2cm}} \text{ with Percent Error} = \underline{\hspace{2cm}}$$

b. Equilibria are found by letting  $P_e = P_{n+1} = P_n$  and  $N_e = N_{n+1} = N_n$  (or setting the right hand side of the differential equation equal to 0), then solving the two algebraic equations. Find all equilibria for this model. The coexistence equilibrium with positive values for both populations is:

$$P_{1e} = \underline{\hspace{2cm}} \text{ and } N_{1e} = \underline{\hspace{2cm}}$$

The other equilibrium satisfies

$$P_{2e} = \underline{\hspace{2cm}} \text{ and } N_{2e} = \underline{\hspace{2cm}}$$

c. In your lab report, create a graph showing the data and the model. How well does the model fit the data? Briefly describe how the predator population responds to rising and declining prey population.

d. Now consider a situation where a toxin enters the water. If it equally kills the predator and the prey proportional to their population, then the revised model becomes:

$$\frac{dP}{dt} = a_1P - a_2PN - \delta_1P$$

$$\frac{dN}{dt} = -b_1N + b_2PN - \delta_1N$$

with  $\delta_1 = 0.06$ . Again use Euler's method to simulate this revised model (with  $h = 0.1$ ). Assume all of the other parameters, including  $P(0)$  and  $N(0)$  are the same as you found in Part a.

From the simulation, estimate the number of *Paramecium caudatum*,  $P(t_i)$ , and *Didinium nasutum*,  $N(t_i)$ , when  $t = 2, 10$ , and 12 days.

$$\text{At } t = 2 \text{ days, } P(2) \approx \underline{\hspace{2cm}} \text{ and } N(2) \approx \underline{\hspace{2cm}}$$

$$\text{At } t = 10 \text{ days, } P(10) \approx \underline{\hspace{2cm}} \text{ and } N(10) \approx \underline{\hspace{2cm}}$$

$$\text{At } t = 12 \text{ days, } P(12) \approx \underline{\hspace{2cm}} \text{ and } N(12) \approx \underline{\hspace{2cm}}$$

e. Again equilibria are found by letting  $P_e = P_{n+1} = P_n$  and  $N_e = N_{n+1} = N_n$  (or setting the right hand side of the differential equation equal to 0), then solving the two algebraic equations.

Find all equilibria for this model. The coexistence equilibrium with positive values for both populations is:

$$P_{1e} = \text{_____} \text{ and } N_{1e} = \text{_____}$$

The other equilibrium satisfies

$$P_{2e} = \text{_____} \text{ and } N_{2e} = \text{_____}$$

f. In your lab report, create a graph showing the simulations of these predator-prey models with and without the toxin.

(Don't include the data on this graph.) Based on the equilibrium values, what is the effect of the toxin on the predator and the prey populations as compared to the equilibrium populations without the toxin? Briefly discuss what are the ramifications of applying pesticide to an agricultural system where there is an agricultural pest (prey) and its natural predator, both sensitive to the pesticide.