1. **(1 pt) mathbioLibrary/setABioc2Labs/Lab122_J1_log_nonauto_country.pg**

Because of the accuracy of WebWork, you should use 5 or 6 significant figures on this problem.

The growth rates of the population for most countries around the world are declining over the past decades. A few countries have actually begun to decline in population. In this problem we examine two models of population growth for Columbia over the last half century. We review our previous work on the continuous logistic population model, then we extend our work to the time varying Malthusian growth model using a linearly declining growth rate.

Below is a Table with approximate population data (in millions) for Columbia from 1950 to 2000.

Year	Pop (M)	Year	Pop (M)
1950	11.57	1980	26.54
1960	15.97	1990	32.81
1970	21.37	2000	39.72

a. One of the most common models used in biology is the continuous logistic growth model which is given by the differential equation:

$$\frac{dP}{dt} = rP\left(1 - \frac{P}{M}\right), \qquad P(0) = P_0,$$

with parameters r and M and initial condition P_0. Find the general solution of this differential equation. Write the solution with 'P0' for P_0, 'M' for M, and 'r' for r.

$P(t) =$ _____

Use Excel's Solver to find the best values of parameters P_0, M, and r to fit the population data for Columbia. Assume that $t = 0$ **corresponds to 1950** and is in years. Include the sum of squares error. Also, write the complete formula with the best parameters fit to the model.

$P_0 =$ _____

$r =$ _____

$M =$ _____

$P(t) =$ _____

$SSE =$ _____

What does this model predict will be the carrying capacity of Columbia? Carrying Capacity = _____

b. Human populations are affected by changes in health, technology, and education. These factors have a time-varying effect on growth rate more than a density dependent growth rate as seen in the logistic growth model. A Malthusian growth model with a time-varying growth rate provides a good fit to this situation where the population growth is declining. The nonautonomous Malthusian growth model is given by the following differential equation:

$$\frac{dP}{dt} = (b - at)P, \qquad P(0) = P_0,$$

with parameters a and b and initial condition P_0. This model has a simple decreasing linear growth rate with Malthusian growth rate b. Find the general solution of this differential equation. Write the solution with 'P0' for P_0, 'a' for a, and 'b' for b.

$P(t) =$ _____

Use Excel's Solver to find the best values of parameters P_0, a, and b to fit the population data for Columbia. Again assume that $t = 0$ **corresponds to 1950** and is in years. Include the sum of squares error. Also, write the complete formula with the best parameters fit to the model.

$P_0 =$ _____

$a =$ _____

$b =$ _____

$P(t) =$ _____

$SSE =$ _____

With the declining growth rate, there is a point in time when the population of Columbia achieve a maximum according to this model, then starts declining. Find the value of t when Columbia has its maximum population according to this model and determine its population at that time. What year does this occur?

$t_{max} =$ _____ yr

$P(t_{max}) =$ _____

Occurs in the year = _____

c. Use the two models to predict the population of Columbia in 2010, 2025, and 2050.

Population from logistic growth model in 2010 = _____

Population from logistic growth model in 2025 = _____

Population from logistic growth model in 2050 = _____

Population from nonautonomous Malthusian growth model in 2010 = _____

Population from nonautonomous Malthusian growth model in 2025 = _____

Population from nonautonomous Malthusian growth model in 2050 = _____

e. In your lab report, create a graph (in Excel) showing the population data for Columbia, the logistic growth model, and the nonautonomous Malthusian growth model. Discuss how well each model fits the data. Which model do you consider to be the better model? Explain a few strengths and weaknesses of each model.