1. (1 pt) mathbioLibrary/setABioc2Labs/Lab122_H2_Lead_Exposure.pg
Because of the accuracy of WebWork, you should use 5 or 6 significant figures on this problem.

For many years, lead (Pb) was an additive to paint used to reduce molds and improve adhesion. Lead was also a gas additive used to improve combustion and reduce the knocking of car engines under stress. These sources have created a major problem of lead-laden dust, especially in the inner city. Lead is problematic in the neural development of small children. Small children are exposed to lead through the dust ingested by normal hand-to-mouth play activities and from breathing the lead-laden dust. Once the lead enters the body it does not leave the body (or leaves very slowly). This lead builds up in the children's bodies and has been linked to developmental problems with their nervous system. Scientists have discovered that lead concentrations as low as 5 mg/dl in the blood results in developmental toxicity. (This is a new level as of 2012, down from 10 mg/dl in the blood earlier.) (**Lead in Children**)

a. The exposure of lead for children begins very low (since small babies hardly move), then increase to maximum during the early years from crawling and hand-to-mouth activities. As the child increases in height, he or she moves further away from the primary contaminated source and drops many of the hand-to-mouth activities, which lowers exposure. Assume that the weighted activity that exposes a boy to lead as a function of the age, t, in years satisfies the differential equation:

$$\frac{dA}{dt} = -kA + be^{-qt}, \qquad A(0) = 0,$$

where $A(t)$ is the weighted activity time of exposure in hours per day. Suppose that the values of the parameters are $k = 0.42$ (yr^{-1}), $b = 5.2$ (hr/day/yr), and $q = 0.7$ (yr^{-1}). Solve this differential equation. Find the weighted activity time of exposure at ages 1, 5, and 9.

$A(t) =$ ______________________ .
Activity at 1, $A(1) =$______ hr/day.
Activity at 5, $A(5) =$______ hr/day.
Activity at 9, $A(9) =$______ hr/day.

Find the maximum level of activity exposing the boy to lead and the age at which this occurs.
Age of Maximum Activity $t_{max} =$ ______ yr.
Maximum Activity Time of Exposure $A(t_{max}) =$ ______ hr/day.

b. In your Lab Report, create a graph for the weighted activity time of exposure, $A(t)$, for $t \in [0, 12]$. Briefly discuss if this graph reasonably models lead exposure for young children based on your understanding of child behavior and where the lead persists.

c. The lead enters the boy's body proportional to his weighted activity time, $A(t)$, and is assumed to not leave following exposure. This suggests that the total amount of lead, $P(t)$, in his body (in μg) will satisfy the following differential equation:

$$\frac{dP}{dt} = KA(t), \qquad P(0) = 0,$$

where $A(t)$ is the solution obtained from Part a and $K = 600$ μg day/hour of play/yr. Find the solution $P(t)$ and determine the amount of lead in the body at ages 1, 5, and 9.

$P(t) =$ ______________________ .
Amt of Pb at 1, $P(1) =$______ μg.
Amt of Pb at 5, $P(5) =$______ μg.
Amt of Pb at 9, $P(9) =$______ μg.

d. In your Lab Report, graph this solution for accumulation of lead in the boy for $t \in [0, 12]$. Briefly discuss this graph and explain how well this differential equation models the accumulation of lead in a child.

e. We have seen that the von Bertalanffy equation of growth provides an approximation for the weight gain of a child. Assume that the boy grows according to the initial value problem,

$$\frac{dw}{dt} = r(73 - w), \qquad w(0) = 2.8,$$

where $r = 0.064$. Find the solution $w(t)$. Based on this model, determine the weight of the boy at ages 1, 5, and 9. What would be the maximum weight of this boy for large values of t? (Note that the equation for $w(t)$ loses accuracy significantly through the teenage years.)

$w(t) =$ ______________________ .
Weight of boy at 1, $w(1) =$______ kg.
Weight of boy at 5, $w(5) =$______ kg.
Weight of boy at 9, $w(9) =$______ kg.
Maximum Weight of boy, $w_{max} =$______ kg.

e. In your Lab Report, graph this solution for the weight of the boy for $t \in [0, 12]$. Briefly discuss this graph and explain how well this differential equation models the weight of a child.

f. Assume that this lead is uniformly distributed throughout the body. If the concentration of lead in the blood (in μg/dl), $c(t)$, satisfies the equation,

$$c(t) = \frac{0.1P(t)}{w(t)},$$

Find the concentration of lead in the blood of the boy at ages 1, 5, and 9. What would be the maximum concentration of lead in the blood of this boy and at what age does this occur.

Concentration of lead in boy at 1, $c(1) =$______ μg/dl.
Concentration of lead in boy at 5, $c(5) =$______ μg/dl.
Concentration of lead in boy at 9, $c(9) =$______ μg/dl.
Maximum Concentration of lead in boy, $c_{max} =$______ μg/dl.
Age of Maximum Concentration $t_{max} =$______ yr.

g. In your Lab Report, graph this solution for the concentration of lead in the boy for $t \in [0, 12]$. Briefly discuss this graph and explain how well this differential equation models the concentration of lead in a child. Check the website given above or any other sources and write a brief paragraph describing the health risks that the boy modeled above might encounter.