

1. (1 pt) mathbioLibrary/setABioc2Labs/Lab122_G3.drug.pg

Because of the accuracy of WebWork, you should use 5 or 6 significant figures on this problem.

Several of you are considering careers in medicine and biotechnology. Drug therapy and dose response is very important in the treatment of many diseases, particularly cancer. Since cancer cells are very similar to your normal body cells, their destruction relies on very toxic drugs. There are some very fine lines in certain cancer treatments between an ineffective dose, one that destroys the cancer, and one that is toxic to all cells in the body. At the base of many of the calculations for these treatments are simple mathematical models for drug uptake and elimination.

a. The simplest situation calls for an injection of the drug into the body. In this case, the differential equation describing the amount of drug in the body is given by:

$$\frac{dA}{dt} = -kA, \quad A(0) = A_0,$$

where A_0 is the amount of drug injected and k depends on how the drug is metabolized and excreted from the body. Suppose that $A_0 = 18 \mu\text{g}$ of a particular drug is injected into the body, and that it has been determined that the half-life of the drug in this patient is 23 days. Solve this differential equation and find the value of k .

$$k = \underline{\hspace{2cm}}.$$

Solution of the differential equation, using the value of k .

$$A(t) = \underline{\hspace{2cm}}.$$

Determine how long the drug is effective, if it has been determined that the patient must have $4.4 \mu\text{g}$ in his body.

$$\text{Effective for } t_e = \underline{\hspace{2cm}} \text{ days.}$$

b. With new materials being developed, the drug can be inserted into polymers that slowly decay and release the drug into the body (See **Norplant**). This delivery system can prevent large toxic doses in the body and maintain the drug level for longer at therapeutic doses. A differential equation that describes type of drug delivery system is given by

$$\frac{dA}{dt} = r e^{-qt} - kA, \quad A(0) = 0,$$

where $r = 1.98 \mu\text{g}/\text{day}$, $q = 0.11 (\text{day}^{-1})$, and k is from Part a, since this is the same drug, which is being metabolized and

excreted similarly. (It can be shown with integration that if $r/q = A_0$, then this is the same amount of drug as delivered in Part a.) Solve this differential equation.

$$A(t) = \underline{\hspace{2cm}}.$$

Over what time period (if any) is this therapy effective. Is this time period longer or shorter than your answer from Part a?

$$\text{Effective for } t_e = \underline{\hspace{2cm}} \text{ days.}$$

LONGER or SHORTER

Find the time, t_{max} , where the maximum occurs and the value of $A(t_{max})$.

$$t_{max} = \underline{\hspace{2cm}}$$

$$A(t_{max}) = \underline{\hspace{2cm}}$$

c. In your Lab Report, create a single graph that shows both solutions for 100 days ($t \in [0, 100]$.) Briefly, describe the graphs of these treatments and what are the significant differences. Which treatment do you consider to be superior and why?

d. For this part of the problem, we want to find the numerical solution of the differential equation in Part b, using the Improved Euler's method of the previous lab. Take a stepsize of $h = 0.5$ on the differential equation describing the drug delivery system with polymers and use the Improved Euler's method to simulate the differential equation for $t \in [0, 100]$. Below enter the solutions at times $t = 12, 40$ and 98 for the model found in Part b, the solution found using Improved Euler's Method and the percent error between these two values.

For time $t = 12$:

$$\text{Actual Solution } \underline{\hspace{2cm}} \mu\text{g}$$

$$\text{Improved Eulers } \underline{\hspace{2cm}} \mu\text{g}$$

$$\text{Percent Error } \underline{\hspace{2cm}}$$

For time $t = 40$:

$$\text{Actual Solution } \underline{\hspace{2cm}} \mu\text{g}$$

$$\text{Improved Eulers } \underline{\hspace{2cm}} \mu\text{g}$$

$$\text{Percent Error } \underline{\hspace{2cm}}$$

For time $t = 98$:

$$\text{Actual Solution } \underline{\hspace{2cm}} \mu\text{g}$$

$$\text{Improved Eulers } \underline{\hspace{2cm}} \mu\text{g}$$

$$\text{Percent Error } \underline{\hspace{2cm}}$$

e. In your Lab Report, create an Excel graph of the numerical solution using the Improved Euler's method with the actual solution found in Part b. Does this numerical solution adequately represent the actual solution of the differential equation?