1. (1 pt) mathbioLibrary/setABioc2Labs/Lab122_G2_Newton_cool_cat.pg

Because of the accuracy of WebWork, you should use 5 or 6 significant figures on this problem.

a. The normal body temperature for cats varies over a narrow range of temperatures. One cat has a history of a body temperature of 39.1°C. One night this cat is hit by a car at some time during the night. The dead cat is discovered at 7 AM, and a young scientist wants to determine the time of death of the poor cat. He measures the temperature of the cat and finds that the body temperature of the cat is 22.6°C ($H(0) = 22.6$). For this problem, $t = 0$ corresponds to 7 AM. The early morning temperature is found to be 15°C. Newton's Law of Cooling with a constant environmental temperature ($T_e = 15$) gives the differential equation:

$$\frac{dH}{dt} = -k_1(H-15), \qquad H(0) = 22.6$$

where $H(t)$ is the body temperature of the cat and k_1 is a kinetic constant of cooling. Find the solution to this differential equation with its initial condition (including the cooling constant k_1 written as 'k1').

$H(t) =$ ______ .

If one hour later the temperature of the body is found to be 20.6°C ($H(1) = 20.6$), then determine the value of the constant of cooling, k_1, in the differential equation.

$k_1 =$ ______ .

Find the time t_d when the death occurs and give the time on the clock.

$t_d =$ ______ .

Time on the clock = ______ : ______ AM,
where the minutes are a decimal value with at least 4 significant figures.

b. Since it is early in the morning, the temperature has been decreasing for some length of time rather than remaining constant. Suppose that a more accurate cooling law is given by the differential equation

$$\frac{dH}{dt} = -k_2(H-(15-0.4t)), \qquad H(0) = 22.6$$

where $H(t)$ is the body temperature of the cat and k_2 is a kinetic constant of cooling. Find the solution to this differential equation with its initial condition (including the cooling constant k_2 written as 'k2').

$H(t) =$ ______ .

Determine the value of the constant of cooling, k_2, in this differential equation.

$k_2 =$ ______ .

Find the time t_d when the death occurs assuming this linear environmental temperature and give the time on the clock.

$t_d =$ ______ .

Time on the clock = ______ : ______ AM,
where the minutes are a decimal value with at least 4 significant figures.

c. In the trigonometric section, we found that the daily temperature is often well approximated by a trigonometric function. Suppose that the environmental temperature is approximated by the function:

$$T_e(t) = 16.6 - 2.4\cos(\omega(t+3)),$$

where $t = 0$ is 7 AM and $\omega = 0.2618$. This temperature function gives the environmental temperature at 7 AM as:

Temperature at 7 AM = ______ .

The minimum temperature has what value and occurs what time during the night.

Minimum temperature = ______ .

Clock Time of Min Temp = ______ : ______ AM,
where the minutes are a decimal value with at least 4 significant figures.

If the environmental temperature follows the trigonometric function given by $T_e(t)$, then a more accurate cooling law is given by the differential equation:

$$\frac{dH}{dt} = -k_3(H - T_e(t)), \qquad H(0) = 22.6$$

where $H(t)$ is the body temperature of the cat and k_3 is a kinetic constant of cooling. Find the solution to this differential equation using Maple, but don't bother to write this solution as you will see that it is rather long and messy. Still you can use similar techniques to the ones above to determine the value of the constant of cooling, k_3, in this differential equation.

$k_3 =$ ______ .

Find the time t_d when the death occurs assuming this trigonometric environmental temperature and give the time on the clock.

$t_d =$ ______ .

Time on the clock = ______ : ______ AM,
where the minutes are a decimal value with at least 4 significant figures.

d. In your Lab Report, create a graph showing the three different environmental temperatures for $t \in [-7, 2]$ (from midnight to 9 AM). Compare and contrast these graphs. Discuss how each of these environmental temperature graphs coincide with your understanding of daily temperature during this period of time.

On a different graph, show the three different environmental temperatures and add the body temperature of the cat for $t \in [-7, 2]$ (from midnight to 9 AM). Be sure to make the body temperature of the cat equal to 39.1°C up until the time of death. Include data points for the body temperature at 7 and 8 AM. Briefly, discuss the differences between the predictions and how accurate you believe these predictions to be. Do the more complicated environmental temperature approximations predict a significantly different time of death?