1. **(1 pt) mathbioLibrary/setABioc2Labs/Lab122_F4_ecoli_grow.pg**
Because of the accuracy of WebWork, you should use 5 or 6 significant figures on this problem.

The shape of a cell affects its surface area to volume ratio. This can be significant in the cell's ability to absorb nutrients and oxygen, remove wastes or survive toxins. Recall that the volume and surface area of a sphere of radius r are given by

$$V_{sphere} = \frac{4}{3}\pi r^3 \qquad SA_{sphere} = 4\pi r^2,$$

while the volume and surface area of a cylinder of radius r and height h are given by

$$V_{cylinder} = \pi r^2 h \qquad SA_{cylinder} = 2\pi r(h+r).$$

The volume and surface area of a prolate ellipsoid (egg-shaped) with its circular radius, a, and longer radius (half the length of the ellipsoid), b, are given by:

$$V_{ellipsoid} = \frac{4}{3}\pi a^2 b \qquad SA_{ellipsoid} = 2\pi\left(a^2 + \frac{ab^2 \arccos(a/b)}{\sqrt{b^2-a^2}}\right).$$

a. This problem begins by examining a number of different organisms with different cellular geometries and having you compute the Volume, Surface Area, and Surface Area to Volume ratio. (Note that the problem gives the diameter of the cell and not the radius.)

We begin with one of the smallest organisms, a *Mycoplasma* . This organism is a parasite or commensal, growing in animal hosts, having small genomes, and lacking a cell wall. They are spherical in shape and have a diameter of $d = 0.54$ μm. Determine the Volume, Surface Area, and Surface Area to Volume ratio. Write the Surface Area to Volume ratio as the fraction SA/V.

Volume, $V =$ _____ μm^3.
Surface Area $SA =$ _____ μm^2.
Ratio $SA/V =$ _____ .

Next we consider *Staphylococcus* , which are gram-positive bacteria that often reside on skin and mucous membranes. Some species of this bacteria are pathogenic and certain strains have become drug resistant and quite dangerous. All *cocci* bacteria are spherical in shape. Consider *Staphylococcus* that have a diameter of $d = 1$ μm. Determine the Volume, Surface Area, and Surface Area to Volume ratio. Write the Surface Area to Volume ratio as the fraction SA/V.

Volume, $V =$ _____ μm^3.
Surface Area $SA =$ _____ μm^2.
Ratio $SA/V =$ _____ .

Next we consider *Escherichia coli* , which are gram-negative bacteria that are common in the lower intestinal tract. Most strains are normal flora of the gut and benefit the host by providing certain nutrients and vitamins, but some strains are pathogenic and can cause severe problems. *E. coli* is a rod-shaped or cylindrical shape with diameter of $d = 0.7$ μm and length of $h = 2.05$ μm. Determine the Volume, Surface Area, and Surface Area to Volume ratio. Write the Surface Area to Volume ratio as the fraction SA/V.

Volume, $V =$ _____ μm^3.
Surface Area $SA =$ _____ μm^2.
Ratio $SA/V =$ _____ .

Next we consider brewer's yeast *Saccharomyces cerevisiae* . This is a eukaryotic organism that primarily reproduces by asexual budding. It is the primary organism used in baking and brewing. *S. cerevisiae* has an ellipsoid shape with a circular diameter of $d = 5$ μm and length of $h = 8.7$ μm. Determine the Volume, Surface Area, and Surface Area to Volume ratio. Write the Surface Area to Volume ratio as the fraction SA/V.

Volume, $V =$ _____ μm^3.
Surface Area $SA =$ _____ μm^2.
Ratio $SA/V =$ _____ .

Finally, diatoms are beautiful eukaryotic algae that have a unique silicate cell wall. They contribute a large percent of the oceanic production from photosynthesis. The biogenic silica that the cell wall is composed of can form very intricate and beautiful structures and allows these cells to become very large with a variety of shapes. *Nitzschia stellata* is a diatom that is very prevalent in arctic waters. It forms a long cylindrical shape with a circular diameter of $d = 18$ μm and length of $h = 100$ μm. Determine the Volume, Surface Area, and Surface Area to Volume ratio. Write the Surface Area to Volume ratio as the fraction SA/V.

Volume, $V =$ _____ μm^3.
Surface Area $SA =$ _____ μm^2.
Ratio $SA/V =$ _____ .

b. Suppose that *E. coli* satisfies the continuous Malthusian growth equation given by:

$$\frac{dp}{dt} = k_c p, \qquad p(0) = 200,$$

where $p(0)$ is the initial population and the doubling time for the population is 35 min. Find the value of k_c and write the general solution, using this value of k_c.

$k_c =$ _____ .
$p(t) =$ _____ .

Suppose that *S. cerevisiae* satisfies the continuous Malthusian growth equation given by:

$$\frac{dy}{dt} = k_r y, \qquad y(0) = 1700,$$

where $y(0)$ is the initial population and the doubling time for the population is 110 min. Find the value of k_r and write the general solution, using this value of k_r.

$k_r =$ _____ .

$y(t) =$ _____ .

Assume that the Malthusian growth is maintained, then determine how long it takes for the **total surface area** of the *E. coli* and *S. cerevisiae* populations found above to reach 0.5 m^2 and what are the respective populations at that time. (Recall that $\mu m = 10^{-6}$ m.) How many μm^2 are there in 0.5 m^2?

0.5 $m^2 =$ ____ μm^2.

E. coli Time to reach 0.5 $m^2 =$ _____ min

E. coli Population when 0.5 $m^2 =$ _____

S. cerevisiae Time to reach 0.5 $m^2 =$ _____ min

S. cerevisiae Population when 0.5 $m^2 =$ _____

c. Suppose that *E. coli* and *S. cerevisiae* satisfy the continuous Malthusian growth equation and initial conditions given in Part b. Assume that the Malthusian growth is maintained, then determine how long it takes for the **total volume** of the *E. coli* and *S. cerevisiae* populations found above to reach 2.1 cm^3 and what are the respective populations at that time. (Recall that cm $= 10^{-2}$ m.) How many μm^3 are there in 2.1 cm^3?

2.1 $cm^3 =$ ____ μm^3.

E. coli Time to reach 2.1 $cm^3 =$ _____ min

E. coli Population when 2.1 $cm^3 =$ _____

S. cerevisiae Time to reach 2.1 $cm^3 =$ _____ min

S. cerevisiae Population when 2.1 $cm^3 =$ _____

d. Once again, suppose that *E. coli* and *S. cerevisiae* satisfy the continuous Malthusian growth equation and initial conditions given in Part b. Assume that the Malthusian growth is maintained, then determine how long it takes for the **total volume** of the *E. coli* and *S. cerevisiae* populations to be equal. Also, find the populations of each organism at this time. (If this never happens, then write 'NEVER' in all three spaces below.)

Volumes Equal at $t_e =$ _____ min

E. coli Population $p(t_e) =$ _____

S. cerevisiae Population $y(t_e) =$ _____

e. In your Lab Report, create a graph showing the growth of the **total volumes** of *E. coli* and *S. cerevisiae* , assuming these populations satisfy the continuous Malthusian growth equation and initial conditions given in Part b. Make your time interval sufficiently large to include the time when the **volumes** are equal. (If your volumes are never equal, then graph the volumes of the two populations for $t \in [0, 1000]$.) Write a short paragraph about the differences in the **surface area to volume ratio** between the different organisms listed in Part a. Discuss how this ratio would affect growth rate. Give some evolutionary advantages and disadvantages according to the sizes given in Part a.

f. Michael Crichton in the *Andromeda Strain* (1969) states that

"A single cell of the bacterium E. coli would, under ideal circumstances, divide every twenty minutes... [I]t can be shown that in a single day, one cell of *E. coli* could produce a supercolony equal in size and weight to the entire planet Earth."

The Earth is not a perfect sphere (it is an oblate spheroid) and has some variance in its radius. For this problem, assume that the Earth is a perfect sphere and take the radius of the Earth to be $r = 6375$ km. Find the volume of the Earth in μm^3.

Volume of Earth = _____ μm^3

E. coli has the ability to grow very rapidly, for this part of the problem assume that *E. coli* can double its population every 17 min. Use the Malthusian growth model in Part b with $p(0) = 1$. Find the new value of k_c with *E. coli* doubling every 17 min and write the new Malthusian growth model starting with a single *E. coli* and using this k_c.

$k_c =$ _____ .

$p(t) =$ _____ .

With this information and the **volume** that you found for *E. coli* above, find how long it takes for a single *E. coli* with a doubling time of 17 to occupy the entire **volume** of Earth.

Time to Fill Earth = _____ .

Was Michael Crichton correct? (Yes or No) _____ .

g. Write a brief paragraph about exponential growth of populations. What do your calculations in the previous part of the problem tell you about the Malthusian growth model?