

**1. (1 pt) mathbioLibrary/setABioc2Labs/Lab122\_A1\_opt\_vol.pg**

Because of the accuracy of WebWork, you should use 5 or 6 significant figures on this problem.

This problem is a classic problem in optimization from Calculus (with a couple new twists). Consider a rectangular piece of cardboard that is  $L = 96$  cm long and  $W = 72$  cm wide. You create an open box by cutting square corners ( $x$  cm by  $x$  cm) from each of the four corners and bending up the sides. This study examines how the volume and surface area of this open box varies as you vary the size of the four corners that you cut out.

a. Write a function for the volume  $V(x)$  of your open box depending on the size  $x$  of the square corners that you cut out.

$$V(x) = \underline{\hspace{2cm}} .$$

What is the domain of this function (from practical physical constraints)?

$$\text{Minimum } x = \underline{\hspace{2cm}} \text{ cm and maximum } x = \underline{\hspace{2cm}} \text{ cm}$$

Compute the derivative of  $V(x)$ .

$$V'(x) = \underline{\hspace{2cm}} .$$

Find the maximum volume and how large a corner you must cut to create this volume.

$$\text{Maximum Volume} = \underline{\hspace{2cm}}$$

$$\text{Value of } x \text{ at Max Volume} = \underline{\hspace{2cm}} \text{ cm}$$

b. What is the area of the paper with no corner's cut out?

$$\text{Area} = \underline{\hspace{2cm}} .$$

Write a function for the surface area  $S(x)$  of your open box depending on the size  $x$  of the square corners that you cut out.

$$S(x) = \underline{\hspace{2cm}} .$$

Give the domain and range of this function.

$$\text{Minimum } x = \underline{\hspace{2cm}} \text{ cm and maximum } x = \underline{\hspace{2cm}} \text{ cm}$$

Find the absolute maximum and absolute minimum for the surface area  $S$ .

$$\text{Minimum } S = \underline{\hspace{2cm}} \text{ and maximum } S = \underline{\hspace{2cm}}$$

Is this function only increasing or decreasing (monotonic)?

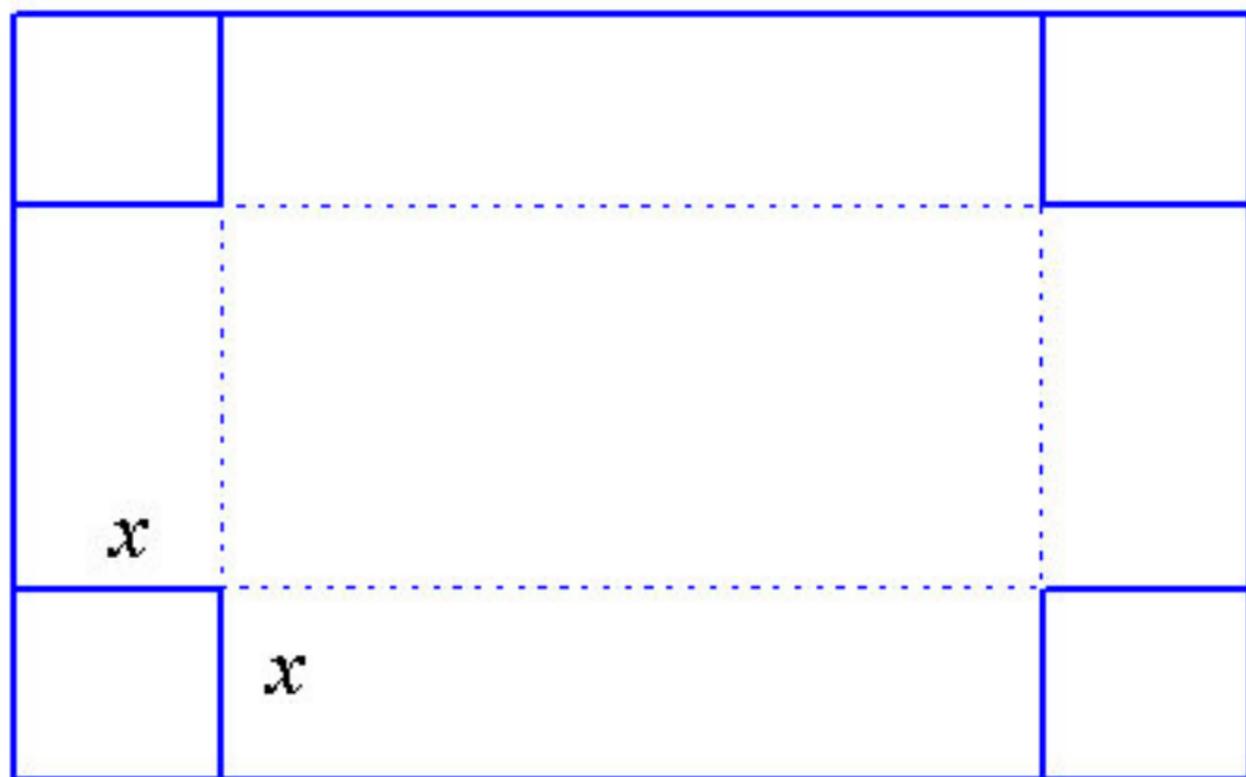
This function is (Increasing, Decreasing, or Neither) \_\_\_\_\_

Find the value of  $S$  when  $V$  is maximum.

$$\text{At } V_{max}, S = \underline{\hspace{2cm}}$$

c. In your Lab report, create a graph  $V(x)$  on its domain. Next make a second graph  $S(x)$  on its domain. Finally, we plot a third graph of the volume as a function of the surface area,  $V(S)$ . This last graph can be created without first solving one of these equations for  $x$ . Explain why this can be done. Write a brief description for each of the graphs that you have created. Describe the shape of the box near the limits of the domain.

**Diagram for Problem**

$L$  $W$  $x$  $x$