Calculus for the Life Sciences II

Lecture Notes – Introduction

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Fall 2012

Outline

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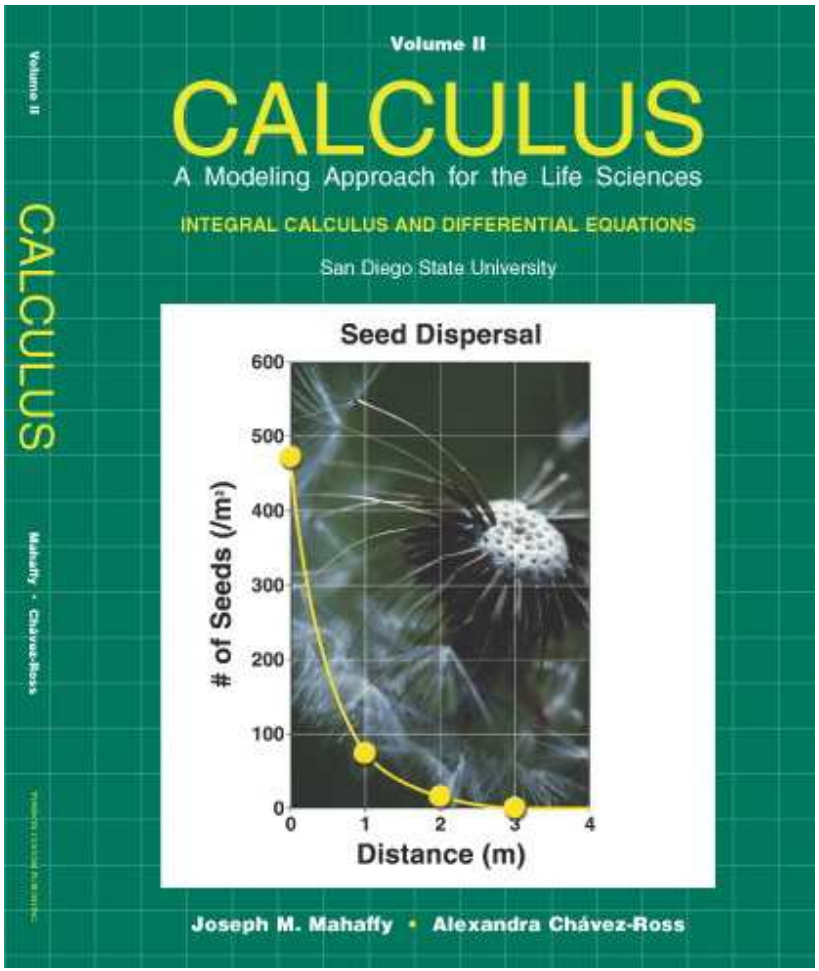
Office Hours	1-2 MW and 3-4 MW, and by appointment

TA Contact Information

TA	Vinnie Berardi
Email	**berardi@rohan.sdsu.edu**
Office Hours	1:30-3 TTh in GMCS 425, and by appointment

Basic Information: The Book



Title:
"Calculus: A Modeling Approach for the Life Sciences"
Volume II

Authors:
Joseph M. Mahaffy &
Alexandra Chàvez-Ross

Publisher:
Pearson Custom Publishing

ISBN:
0-536-90522-3

Basic Information: Syllabus

- Review Derivative
- Discrete Dynamical Models
- Optimization
- Trigonometric Functions
- Differential Equations and Integration
  - Linear Differential Equations
  - Numerical Differential Equations
  - Integration
  - Separable Differential Equations
  - Integration by Substitution
  - Riemann Sums/Numerical Integration
  - Definite Integral
  - Integration by Parts
  - Qualitative Analysis of Differential Equations

Basic Information: Grading

Detailed information is found on the
Homework and Assignment Web Page

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- **Lecture Material is 2/3 of grade**
  - Homework with WeBWorK (20% of Lecture grade)
  - 3 Exams (16% each)
  - Final (32%)
  - **Scientific Calculator only** - Exams and Final
  - One 3x5 notecard for Exams and three 3x5 notecards for Final

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- **Lab Work is 1/3 of grade**
  - 8-10 Lab assignments
  - 3 Lab Exams worth twice a regular Lab assignment
    - Open notes, Flashdrive, **no email or cell phone**
    - Use Laptop or assigned computer in GMCS 425 or 422

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  - Please be courteous to other students and the instructor.
  - Abide by university statutes, and all applicable local, state, and federal laws.

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- The instructor will make special arrangements for students with documented learning disabilities and will `try` to make accommodations for other unforeseen circumstances, *e.g.* illness, personal/family crises, etc. in a way that is fair to all students enrolled in the class. ***Please contact the instructor EARLY regarding special circumstances.***

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- Students are expected ***and encouraged*** to ask questions in class!
- Students are expected ***and encouraged*** to to make use of office hours! If you cannot make it to the scheduled office hours: contact the instructor to schedule an appointment!

Expectations and Procedures, III

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  You will receive a **ZERO** for any missed exam, except for **written/documented** excuses (illness, personal/family crises, etc.).

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- **Lab assignments:**
  - Attendance is mandatory or automatic 10 point deduction
  - Partners are assigned and must work with given partner
  - Arriving 20 minutes late or missing a Lab means working the lab alone
  - Labs due promptly by Saturday 4 PM following a given Lab unless told otherwise.
  - Lowest lab score is dropped
  - Your responsibility to back up Lab work – No excuses accepted or extensions granted for lost material

Computer Lab

- Computer Labs are located in GMCS 422 and 425 – Hours are posted on the Lab doors
- Completed Lab Reports are turned into Math 122 box located in GMCS 425
- Software used
  - Excel
  - Word
  - Maple
- Labs are 60% WeBWorK and 40% written report
- **Please direct questions first to your Lab TA**

Math 122: Formal Prerequisites

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Successful Completion of Math 121

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Successful Completion of Math 121

Warning! – If your **Math 121** relied heavily on **Wolfram Alpha**, then you are at a **very distinct disadvantage** in this class

Math 122: Introduction

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- Use real or realistic examples
- Computer labs aid solving more complicated models

Math 122: Introduction — Mathematical Biology

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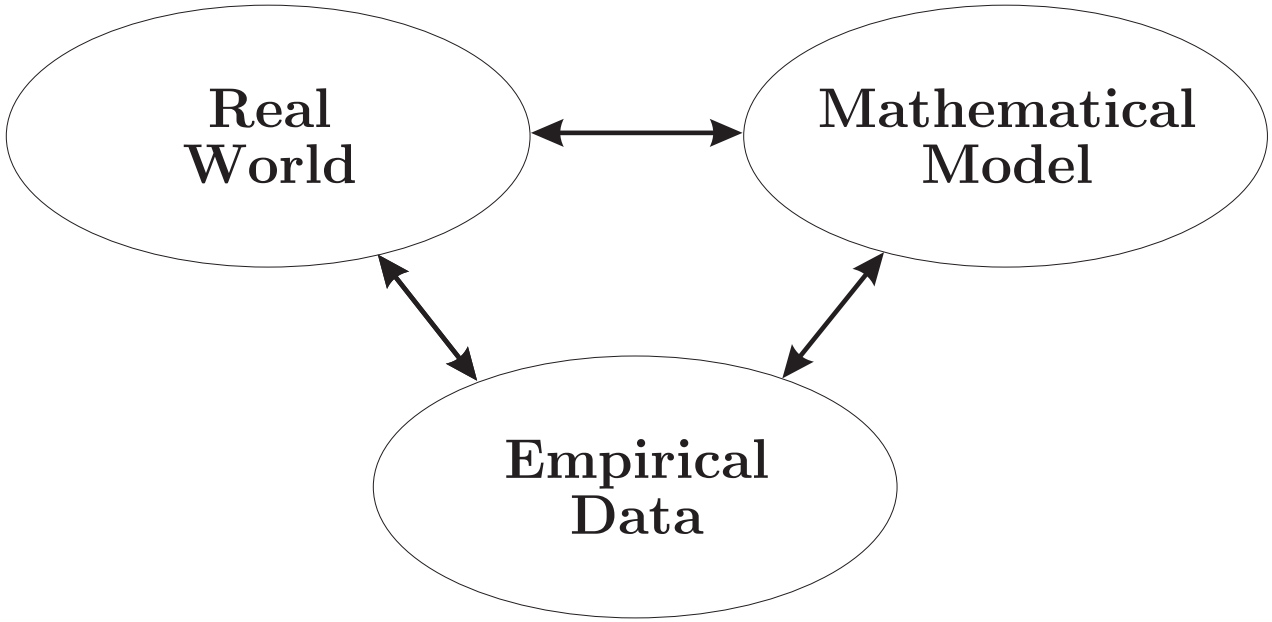
- Biological problems
  - Often stretch mathematical techniques
  - Illustrate mathematical tools well
  - Build intuition for problem solving techniques

Math 122: Introduction — Mathematical Model

So what is a mathematical model?



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- The model may suggest improved experiments

Introduction – Example – Predator-Prey Model 1

Predator-Prey Model

Thanks to Tom and Pat Leeson

Example – Predator-Prey Model 2

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- Widely used by biologists – however, significant flaws in the mathematical understanding often lead to poor conclusions

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Example – Predator-Prey Model

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- We'll examine this model late in the semester

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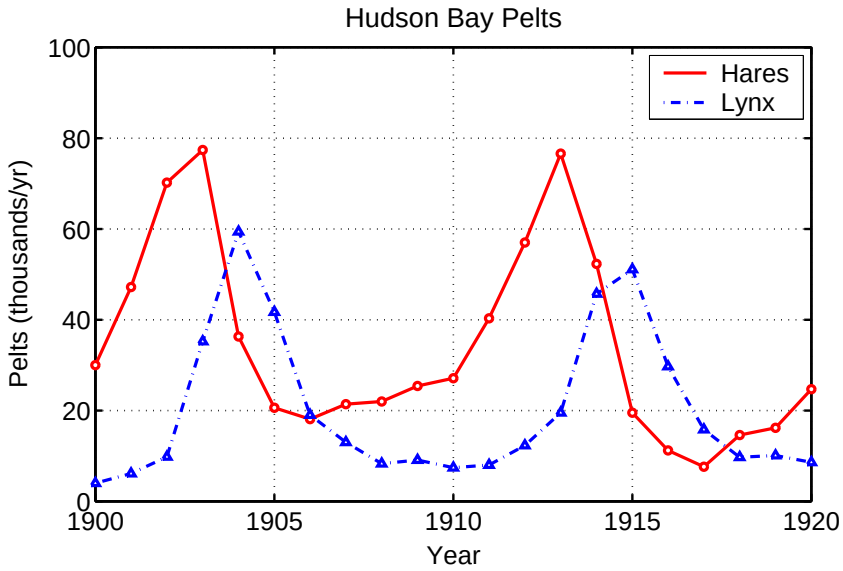

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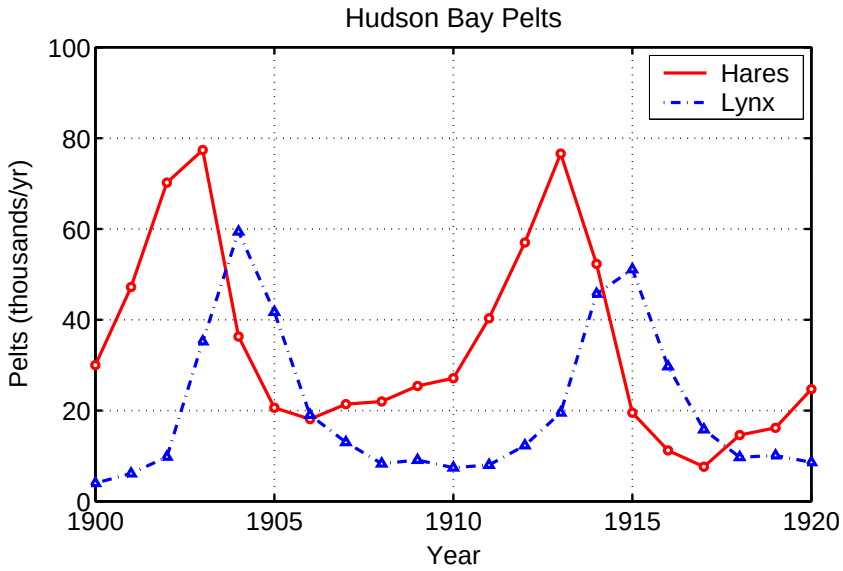

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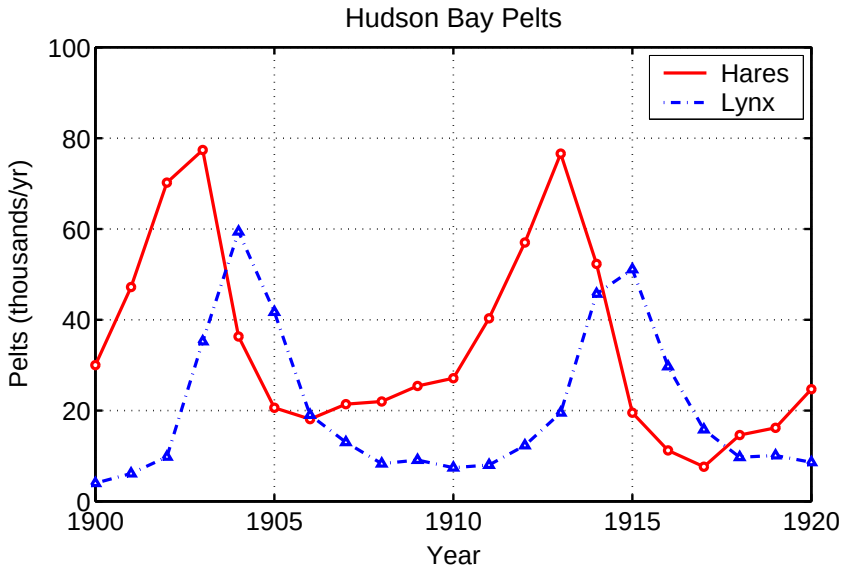

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- Next the hare population plummets, which is followed by lynx population plummeting

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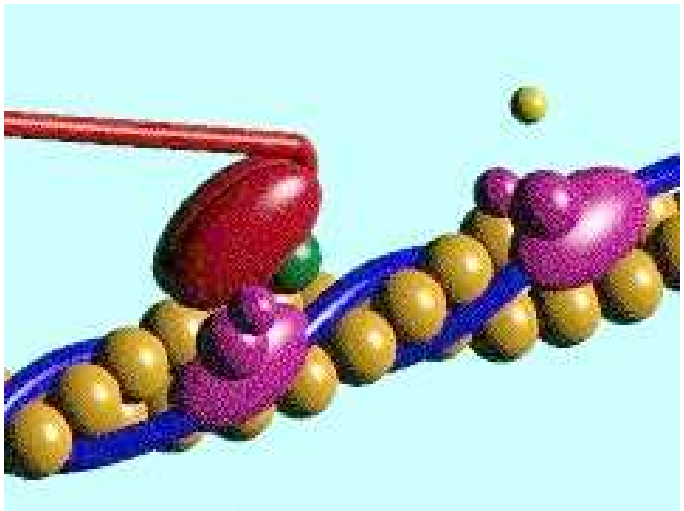
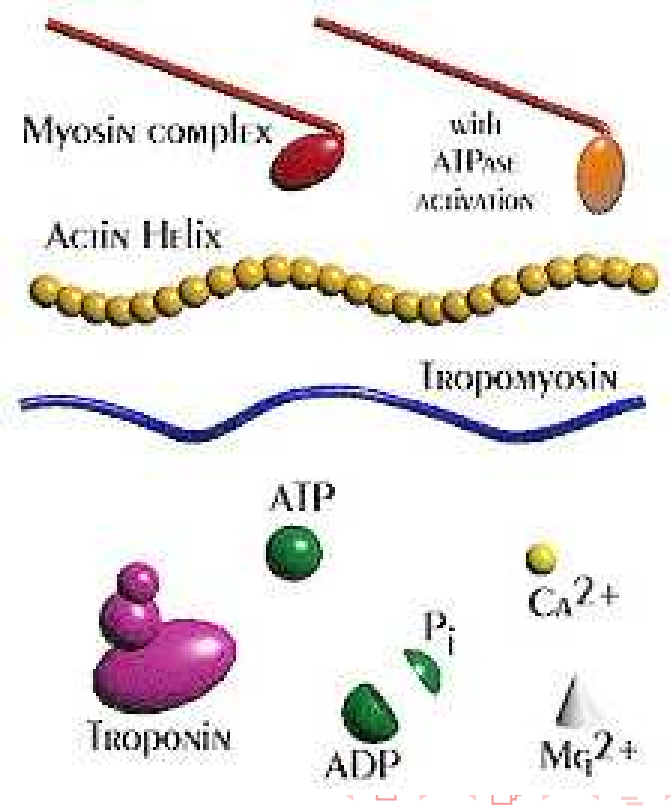
- The force of contracting muscles is generated by chemical reactions that cause a conformational strain in the head of the cross-bridges

Example 2 – Muscle Contraction

The figures show a single cross-bridge projection coming from the myosin filament and indicates how the motion of the head of this cross-bridge element can pull the actin filament toward the center of the cell





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Ratchet Theory of Muscle Contraction

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Cross-bridge Cycle - Biochemical Steps

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- The ATP is hydrolyzed to ADP, which results in a conformational change in the head of the cross-bridge and generates the force to pull the actin filament toward the center of the cell
- Next the ADP is released and binding between the filaments is broken, and the cycle is ready to begin anew with the next binding of ATP

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- A mathematical model for the sliding filament theory is too complicated for this course, but we will be developing some of the basics, such as stretching of an elastic element, which are needed for modeling molecular action of the cross-bridges