Calculus for the Life Sciences II

Lecture Notes – Introduction

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Outline

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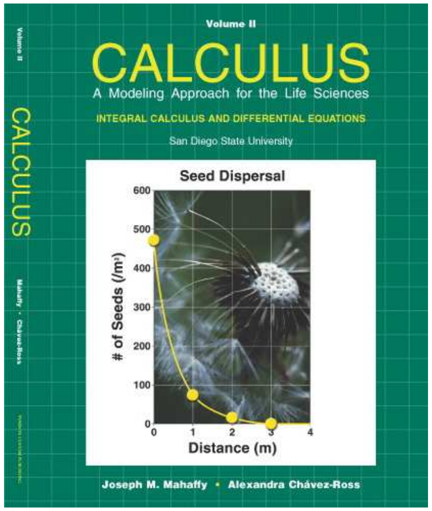
Basic Information: The Book



Title:
"Calculus: A Modeling Approach for the Life Sciences"
Volume II

Authors:
Joseph M. Mahaffy &
Alexandra Chàvez-Ross

Publisher:
Pearson Custom Publishing

ISBN:
0-536-90522-3

Basic Information: Syllabus

- Review Derivative
- Discrete Dynamical Models
- Optimization
- Trigonometric Functions
- Differential Equations and Integration
  - Linear Differential Equations
  - Numerical Differential Equations
  - Integration
  - Separable Differential Equations
  - Integration by Substitution
  - Riemann Sums/Numerical Integration
  - Definite Integral
  - Integration by Parts
  - Qualitative Analysis of Differential Equations

Basic Information: Grading

Detailed information is found on the
Homework and Assignment Web Page

- **Lecture Material is 2/3 of grade**
  - Homework with WeBWorK (20% of Lecture grade)
  - 3 Exams (16% each)
  - Final (32%)
  - **Scientific Calculator only** - Exams and Final
  - One 3x5 notecard for Exams and three 3x5 notecards for Final
- **Lab Work is 1/3 of grade**
  - 8-10 Lab assignments
  - 3 Lab Exams worth twice a regular Lab assignment
    - Open notes, Flashdrive, **no email or cell phone**
    - Use Laptop or assigned computer in GMCS 425 or 422

Expectations and Procedures, I

- Most class attendance is OPTIONAL — Homework and announcements will be posted on the class web page. If/when you attend class:
  - Please be on time.
  - Please pay attention.
  - Please turn off mobile phones.
  - Please be courteous to other students and the instructor.
  - Abide by university statutes, and all applicable local, state, and federal laws.

Expectations and Procedures, II

- WeBWorK assignments are posted with a specific due date. It is **your responsibility** to complete the assignment on time.
- The instructor will make special arrangements for students with documented learning disabilities and will `try` to make accommodations for other unforeseen circumstances, *e.g.* illness, personal/family crises, etc. in a way that is fair to all students enrolled in the class. ***Please contact the instructor EARLY regarding special circumstances.***
- Students are expected ***and encouraged*** to ask questions in class!
- Students are expected ***and encouraged*** to to make use of office hours! If you cannot make it to the scheduled office hours: contact the instructor to schedule an appointment!

Expectations and Procedures, III

- **Missed Exams or Lab Exams: Don't miss Exams!** You will receive a **ZERO** for any missed exam, except for **written/documented** excuses (illness, personal/family crises, etc.).
- **Lab assignments:**
  - Attendance is mandatory or automatic 10 point deduction
  - Partners are assigned and must work with given partner
  - Arriving 20 minutes late or missing a Lab means working the lab alone
  - Labs due promptly by Saturday 4 PM following a given Lab unless told otherwise.
  - Lowest lab score is dropped
  - Your responsibility to back up Lab work – No excuses accepted or extensions granted for lost material

Computer Lab

- Computer Labs are located in GMCS 422 and 425 – Hours are posted on the Lab doors
- Completed Lab Reports are turned into Math 122 box located in GMCS 425
- Software used
  - Excel
  - Word
  - Maple
- Labs are 60% WeBWorK and 40% written report
- **Please direct questions first to your Lab TA**

Math 122: Formal Prerequisites

Math 122: Formal Prerequisites

Successful Completion of Math 121

Warning! – If your **Math 121** relied heavily on **Wolfram Alpha**, then you are at a **very distinct disadvantage** in this class

Math 122: Introduction

- Biology is rapidly expanding - more quantitative analysis of the data
- Mathematics and computers have an increasing role in biology
- This course in Calculus for Biology
  - Emphasis on mathematical modeling of biological systems
  - Lecture notes show how Calculus naturally arises in biological examples
  - Begin with a biological model
  - Mathematical theory required to analyze the biological problem
- Use real or realistic examples
- Computer labs aid solving more complicated models

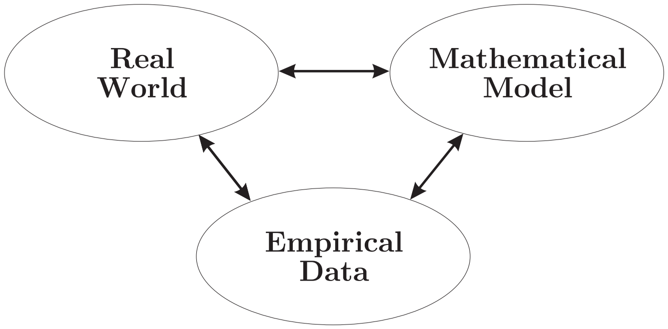
Math 122: Introduction — Mathematical Biology

Mathematical Biology

- Mathematical tools
  - Better qualitative and quantitative understanding of biological problems
  - Suggest alternate possibilities
  - Reject inconsistent ideas
- Biological problems
  - Often stretch mathematical techniques
  - Illustrate mathematical tools well
  - Build intuition for problem solving techniques

Math 122: Introduction — Mathematical Model

So what is a mathematical model?



Math 122: Introduction — Mathematical Model

Mathematical Modeling

- A **mathematical model** is a representation of a real system
- It is simple in design
- It exhibits the basic properties of the real system
- The model should be testable against empirical data
- Comparisons of the model to the real system should lead to improved mathematical models
- The model may suggest improved experiments

Introduction – Example – Predator-Prey Model 1

Predator-Prey Model

Thanks to Tom and Pat Leeson

Example – Predator-Prey Model 2

Predator-Prey Model

- In the early 20^{th} century, Sir Ronald Ross used mathematical modeling to show that malaria could be eliminated without the total eradication of mosquitoes
- A. J. Lotka [1] first studied the population dynamics of predator-prey interactions
- Studies of Vito Volterra on fishing in the Adriatic Sea in 1924 showed value of a simple model for equilibrium analysis
- **Predator-Prey models** are often called **Lotka-Volterra models**
- Widely used by biologists – however, significant flaws in the mathematical understanding often lead to poor conclusions

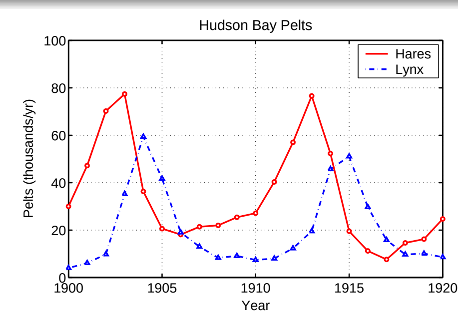
[1] A. J. Lotka (1912), Evolution in Discontinuous Systems, *Journal of the Washington Academy of Sciences*, **2**, pp.2, 49, 66

Example – Predator-Prey Model 3

Classic Lynx-Hare Data

- Records of the Hudson Bay Company show that the pelts of the lynx and hares seemed to oscillate with a fairly regular period
- Simple ecological system, as the lynx is a very specialized predator that primarily feeds on snowshoe hares
- Books often cherry-pick to show limited data - Model fails badly over the complete data set
- We'll examine this model late in the semester

Example – Predator-Prey Model 4



- Graph shows a clear correlation between the populations of lynx and hares
- Rapid rise in the population of the hares is followed by a rapid rise in the lynx population
- Next the hare population plummets, which is followed by lynx population plummeting

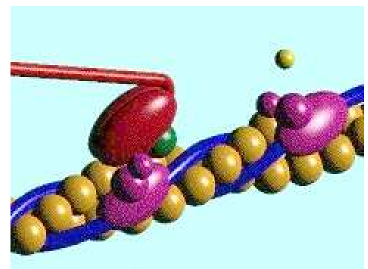
Example 2 – Muscle Contraction 1

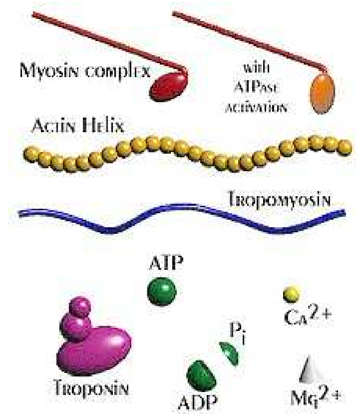
Muscle Contraction

- Muscles take chemical energy in our bodies and convert it into motion
- **Skeletal muscle** is a striated tissue composed of numerous muscle fibers, which each contain hundreds to thousands of myofibrils
- The basic contracting unit of the myofibril is sarcomere, which consists of about 1500 myosin filaments or thick filaments and twice as many actin or thin filaments
- These filaments slide past each other during muscular contraction due to cross-bridges between the two filaments
- The force of contracting muscles is generated by chemical reactions that cause a conformational strain in the head of the cross-bridges

Example 2 – Muscle Contraction 2

The figures show a single cross-bridge projection coming from the myosin filament and indicates how the motion of the head of this cross-bridge element can pull the actin filament toward the center of the cell





Example 2 – Muscle Contraction 3

Ratchet Theory of Muscle Contraction

- The ratchet theory for muscle contraction was first developed by A. F. Huxley in 1957 [2]
- Huxley won a Nobel prize for his model of the giant axon of the squid, which is seminal work in understanding action potentials in nerves
- Biochemical details have been intensely studied

[2] A. F. Huxley (1957), Muscle structure and theories of contraction, *Progress in Biophysics* **7**, 255-318

Example 2 – Muscle Contraction 4

Cross-bridge Cycle - Biochemical Steps

- The **cross-bridge cycle** begins with the binding of ATP to the head of the cross-bridge
- A calcium ion next binds to this protein, which changes the charges and promotes binding of the cross-bridge to the actin filament
- The ATP is hydrolyzed to ADP, which results in a conformational change in the head of the cross-bridge and generates the force to pull the actin filament toward the center of the cell
- Next the ADP is released and binding between the filaments is broken, and the cycle is ready to begin anew with the next binding of ATP

Example 2 – Muscle Contraction

Modeling Process

- This theory provides the framework for many experiments and refinements that continue today to better understand the precise chemical reactions and forces generated by this process
- The process of understanding contraction of a muscle relies on collaborations between biologists studying the muscle tissue, physicists and mathematicians who can calculate the forces generated and required configurations, and chemists who determine key atomic structures, which require massive computational methods
- More on the Physiology can be found in Biology 590
- A mathematical model for the sliding filament theory is too complicated for this course, but we will be developing some of the basics, such as stretching of an elastic element, which are needed for modeling molecular action of the cross-bridges