Calculus for the Life Sciences II

Lecture Notes – Differential Equations and Integration

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  - Time varying buildup of contaminents in an animal

Falling Cat 1

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- Cats evolved a very flexible spine, which aids in their ability to spring for prey, absorb shock from their lightning fast strikes, and rapidly rotate their bodies in mid air
- With their very sensitive inner ear for balance, which is combined with quick reflexes and a flexible spine, a cat that falls is capable of righting itself very rapidly, insuring that it lands on the ground feet first

Falling Cat 2

Falling Cat - Scientific Studies

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- There was a study in the Annals of Improbable Research (1998) on the number of times a particular cat ended up on its feet when dropped from several different heights
- There was a scientific study of cats falling out of New York apartments, where paradoxically the cats falling from the highest apartments actually fared better than ones falling from an intermediate height

[1] Jared M. Diamond (1988), Why cats have nine lives, Nature 332, pp 586-7

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With this information, determine the minimum height from which a cat can be dropped to insure that it lands on its feet

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- Use **Newton's law of motion**
  - **Mass times acceleration is equal to the sum of all the forces** acting on the object
- Since the cat is falling only a short distance at a fairly low velocity, it is safe to assume that the only force acting on the cat is gravity

Falling Cat

Model for the Falling Cat Newton's law of motion

- Equation for **Falling Cat**

$$ma = -mg$$

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  - m is the mass of the cat
  - a is the acceleration of the cat
  - $-mg$ is the force of gravity (assuming up is positive)
- g is a constant ($g = 979$ cm/sec^2 at a latitude like San Diego when you add centripetal acceleration to the standard value given for g, which is 980.7 cm/sec^2)

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Differential Equation

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- This is a **second order linear differential equation**

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$$h(0) = h_0 > 0$$

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- Let ground level be a height of zero, so the cat begins at some height above ground level,

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- The problem reduces to solving this initial value problem and finding what values h_0 of produce a solution such that

$$h(0.3) > 0$$

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- The derivative of a straight line produces a constant
- A straight line with a slope of $-g$ is

$$v(t) = -gt + c$$

Falling Cat

Solution (cont) - Velocity of Cat: The velocity satisfies

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- Recall that a quadratic function has a linear derivative
- The general solution of this initial value problem is

$$h(t) = -\frac{gt^2}{2} + c \qquad c \text{ arbitrary}$$

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$$h(t) = h_0 - 489.5\,t^2$$

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- Evaluate this at $t = 0.3$ sec and obtain

$$h(0.3) = h_0 - 489.5(0.3)^2 = h_0 - 44.055 \text{ cm}$$

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- Thus, the cat must be higher than 44.1 cm for it to have sufficient time to right itself before hitting the ground (This is about 1.5 feet)

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- The solution of the time varying differential equation is written

$$y(t) = \int f(t)\,dt$$

Antiderivatives and Integrals 1

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$$\int t^n dt = \frac{t^{n+1}}{n+1} + C$$

- Note that the integral formula has an arbitrary constant C

Antiderivatives and Integrals

- The integral of a constant is a linear function

$$\int k\,dt = k\,t + C$$

Antiderivatives and Integrals 2

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$$\int \frac{dt}{t} = \ln|t| + C$$

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- **Other Basic Integrals**

$$\begin{aligned} \int e^{kt} dt &= \frac{e^{kt}}{k} + C \\ \int \frac{dt}{t} &= \ln|t| + C \\ \int \cos(kt) dt &= \frac{\sin(kt)}{k} + C \end{aligned}$$

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Antiderivatives and Integrals 3

Linearity of Integrals Let $F(t)$ and $G(t)$ be antiderivatives of $f(t)$ and $g(t)$ and constants k and C

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$$\int (f(t) + g(t))dt = \int f(t)dt + \int g(t)dt = F(t) + G(t) + C$$

- **Scalar Property:**

$$\int k\, f(t)dt = k \int f(t)dt = k\, F(t) + C$$

Integral Example 1

Example 1: Consider the integral

$$\int \left(2\,x^3 - e^{-5x} + \frac{2}{x^2}\right) dx$$

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$$\int \left(2\,x^3 - e^{-5x} + \frac{2}{x^2} \right) dx \quad = \quad 2\left(\frac{x^4}{4} \right)$$

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Solution:

$$\int \left(2\,x^3 - e^{-5x} + \frac{2}{x^2} \right) dx \quad = \quad 2\left(\frac{x^4}{4}\right) - \left(\frac{e^{-5x}}{-5}\right)$$

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Integral Example 2

Example 2: Consider the integral

$$\int \left(4\sin(2x) - \frac{3}{x} \right) dx$$

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$$\int \left(4\sin(2x) - \frac{3}{x}\right) dx \quad = \quad 4\left(\frac{-\cos(2x)}{2}\right) - 3\ln(x) + C$$

Integral Example 2

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$$\int \left(4\sin(2x) - \frac{3}{x} \right) dx$$

Skip Example

Solution:

$$\begin{aligned} \int \left(4\sin(2x) - \frac{3}{x} \right) dx &= 4 \left(\frac{-\cos(2x)}{2} \right) - 3\ln(x) + C \\ &= -2\cos(2x) - 3\ln(x) + C \end{aligned}$$

Integral Example 3

Example 3: Consider the integral

$$\int \left(3\,x^2 - 4\,e^{-x}\right)\,dx$$

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$$\int \left(3\,x^2 - 4\,e^{-x}\right) dx$$

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Solution:

$$\int \left(3\,x^2 - 4\,e^{-x}\right) dx \quad = \quad 3\left(\frac{x^3}{3}\right)$$

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Example 3: Consider the integral

$$\int (3\,x^2 - 4\,e^{-x})\,dx$$

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Solution:

$$\int (3\,x^2 - 4\,e^{-x})\,dx \quad = \quad 3\left(\frac{x^3}{3}\right) - 4(-e^{-x}) + C$$

Integral Example 3

Example 3: Consider the integral

$$\int (3\,x^2 - 4\,e^{-x})\,dx$$

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Solution:

$$\begin{aligned}\int (3\,x^2 - 4\,e^{-x})\,dx &= 3\left(\frac{x^3}{3}\right) - 4(-e^{-x}) + C \\ &= x^3 + 4\,e^{-x} + C\end{aligned}$$

Integral Example 4

Example 4: Consider the integral

$$\int \left(2\,\sin(3x) - \frac{4}{x^3} \right) dx$$

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Solution:

$$\int \left(2\,\sin(3x) - \frac{4}{x^3} \right) dx \quad = \quad 2\left(\frac{-\cos(3x)}{3} \right)$$

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$$\int \left(2\,\sin(3x) - \frac{4}{x^3} \right) dx \;=\; 2\left(\frac{-\cos(3x)}{3} \right) - 4\int x^{-3} dx$$

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$$\begin{aligned}\int \left(2\,\sin(3x) - \frac{4}{x^3} \right) dx &= 2\left(\frac{-\cos(3x)}{3} \right) - 4 \int x^{-3} dx \\ &= -\tfrac{2}{3}\cos(3x) - 4\frac{x^{-2}}{-2} + C\end{aligned}$$

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Integral Example 5

Example 5: Consider the integral

$$\int \left(3\cos\left(\frac{t}{2}\right) - \frac{1}{2t}\right) dt$$

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Integral Example 6

Example 6: Consider the integral

$$\int \left(6\sqrt{t} - e^{2t} - 3\right) dt$$

Skip Example

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$$\int \left(6\sqrt{t} - e^{2t} - 3\right) dt$$

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Solution:

$$\int \left(6\,t^{1/2} - e^{2t} - 3\right) dt$$

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Solution:

$$\int \left(6\,t^{1/2} - e^{2t} - 3\right) dt \quad = \quad 6\left(\frac{t^{3/2}}{\frac{3}{2}}\right)$$

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Differential Equation Example 1

DE Example 1: Initial Value Problem

$$\frac{dy}{dt} = 4\,t - e^{-2t}, \qquad y(0) = 10$$

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Differential Equation Example 2

DE Example 2: Initial Value Problem

$$\frac{dy}{dt} = 2\,t - \sin(t), \qquad y(0) = 3$$

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Differential Equation Example 3

DE Example 3: Initial Value Problem

$$\frac{dy}{dt} = 4 - 2\,y, \qquad y(0) = 5$$

Skip Example

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$$\frac{dy}{dt} = 4 - 2\,y, \qquad y(0) = 5$$

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Solution: Rewrite $y' = -2(y-2)$, so this linear differential equation

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Make substitution $z(t) = y(t) - 2$, so $z(0) = 5 - 2 = 3$

$$\frac{dz}{dt} = -2\,z, \quad \text{with} \quad z(0) = 3$$

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$$z(t) = 3\,e^{-2t} = y(t) - 2 \quad \text{or} \quad y(t) = 2 + 3\,e^{-2t}$$

Differential Equation Example 4

DE Example 4: Initial Value Problem

$$\frac{dy}{dt} = \frac{4}{t}, \qquad y(1) = 2$$

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Solution:

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Height of a Ball: A ball is thrown vertically with gravity ($g = 32$ ft/sec^2) being the only force acting on the ball (air resistence is ignored)

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- Graph the solution for all time up until the ball hits the ground

Height of a Ball 2

Solution: **Newton's law of motion** with the mass of the ball, m, acceleration of the ball, a, and force of gravity, mg, satisfies

$$ma = -mg \qquad \text{or} \qquad a = -g = -32$$

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- The initial velocity is $v(0) = 64$ ft/sec, so

$$v(0) = C = 64 \qquad \text{or} \qquad v(t) = 64 - 32\,t$$

Height of a Ball 3

Solution (cont): **Velocity Equation** satisfies

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Height of a Ball 4

Solution (cont): **Height Equation** is

$$h(t) = 64\,t - 16\,t^2 = -16\,t(t-4)$$

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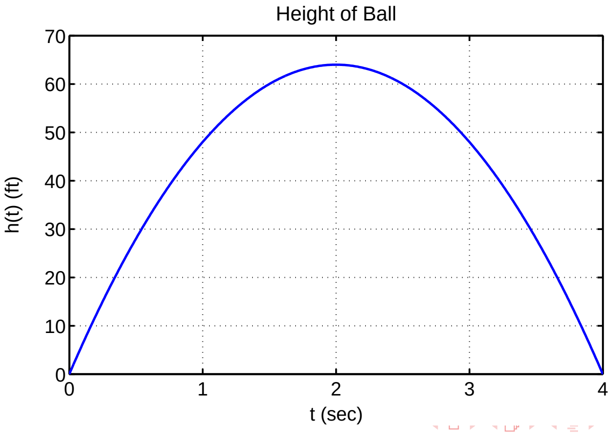
- The maximum height is

$$h(2) = 64(2) - 16(4) = 64 \text{ ft}$$

Height of a Ball

Graph of Height of the Ball



Mercury in Fish 1

Introduction - Fish in the Great Lakes Region

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    - PCBs have declined 90% since 1975

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  - Larger fish consume small fish and concentrate mercury

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Introduction - Mercury and Health

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- The fat problem is for the PCBs, not the Hg

Mercury in Fish

Introduction - Mercury and Health

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- Others are recommended to limit their comsumption to less than one fish a week and avoid eating the larger fish, such as Lake Trout over 22 inches

Mercury in Fish

Introduction - Mercury Buildup in Fish

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- Thus, older and larger fish should have more Hg than the younger fish

Modeling Mercury in Fish

Modeling Mercury in Fish - Model Weight of a Fish

Modeling Mercury in Fish 1

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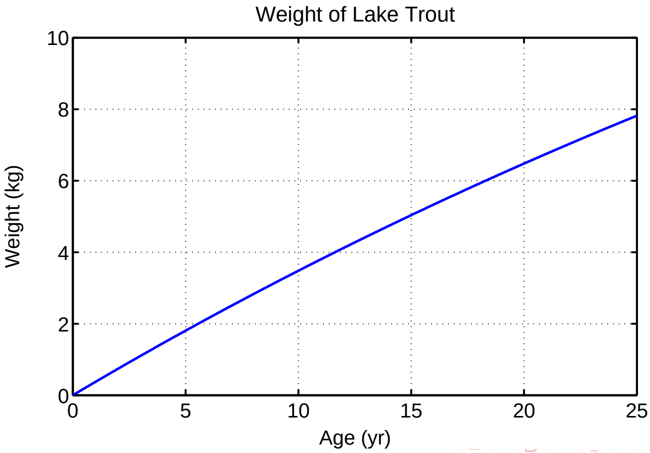
- It takes 5.5 years to reach 2 kg, about 15 years to reach 5 kg, with a limiting level of 25 kg for Lake Trout

Modeling Mercury in Fish 2

Graph for Weight of Lake Trout



Modeling Mercury in Fish

3

Modeling Mercury in Fish - Model for Mercury in a Fish

Modeling Mercury in Fish

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Modeling Mercury in Fish

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- Let $H(t)$ be the amount of Hg (in mg) in a growing Lake Trout
- Assume the concentration of Hg in a fish's food is constant (though in fact, the larger fish will eat more heavily contaminated, larger fish)
- The rate of Hg added to the tissues of a fish satisfies the differential equation with $k = 0.03$ mg/yr:

$$\frac{dH}{dt} = k\left(25 - 25\,e^{-0.015t}\right) \quad \text{with} \quad H(0) = 0$$

Modeling Mercury in Fish 4

Model for Mercury in Fish: The differential equation is

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$$H(t) = k\int \left(25 - 25\,e^{-0.015t}\right) dt$$

- Taking antiderivatives:

$$H(t) = 25k\left(t - \frac{e^{-0.015t}}{-0.015} + C\right)$$

Modeling Mercury in Fish

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Modeling Mercury in Fish 5

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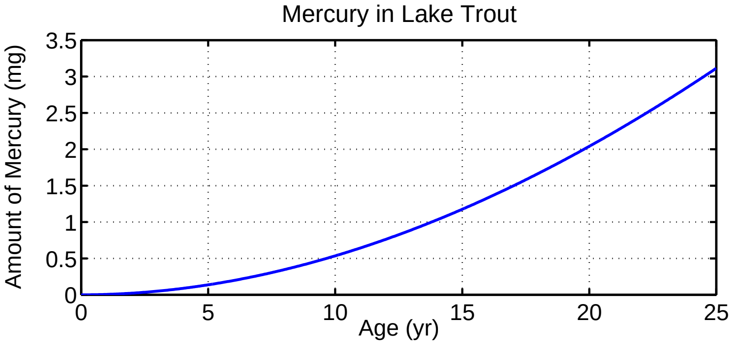
- With $k = 0.03$, the solution becomes

$$H(t) = 0.25(3\,t + 200\,e^{-0.015t} - 200) \text{ mg of Hg}$$

Modeling Mercury in Fish

Graph for Mercury in Lake Trout



Modeling Mercury in Fish

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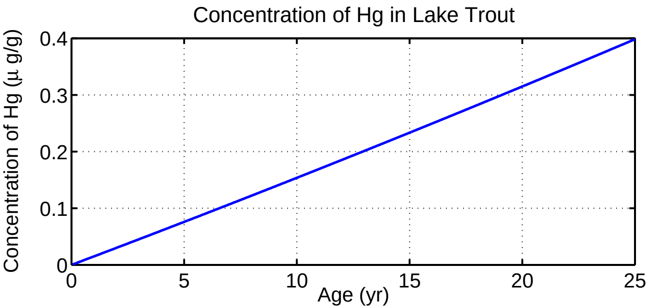
- The amount of mercury in an aging Lake Trout increases as it eats more prey containing mercury
- It is more important to know what the concentration of mercury in the flesh of the Lake Trout that is eaten
- The concentration of mercury increases in a Lake Trout as it ages
- It is clear why the Michigan Department of Health advises against eating larger fish

Modeling Mercury in Fish

Concentration of Mercury in Lake Trout The concentration of Hg is measured in μg (of mercury) per gram (of fish)

$$c(t) = \frac{H(t)}{w(t)}$$



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- As leaded paint is shed or removed, lead enters dust in homes, reaching levels of 0.5% by weight of lead
- Lead-laden dust is particularly problematic in the development of small children
- They are exposed to lead from the dust ingested by normal hand-to-mouth play activities (crawling followed by thumb-sucking or playing with toys and sucking on them)

Lead Build Up in Children 2

Lead Build Up in Children: Neural Effects

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  - Vitamin D metabolism and hemoglobin synthesis is impaired at 40 μg/dl in the blood

Lead Build Up in Children 3

Modeling Lead Build Up in Children

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- Assume that the majority of the lead enters through the ingestion of contaminated dirt
- The amount of lead ingested depends on the hand-to-mouth activity of the child
- This activity usually increases from birth to age 2 years, then rapidly declines as other forms of activity replace the hand-to-mouth activities

Lead Build Up in Children 4

Modeling Activity in a Small Girl

Lead Build Up in Children

Modeling Activity in a Small Girl

- Assume that the hand-to-mouth activity of a small girl in the age 0-24 months satisfies the differential equation

$$\frac{da}{dt} = 0.02(12 - a) \quad \text{with} \quad a(0) = 0$$

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- Graph the solution
- Determine the activity level in hours per day at age 24 months

Lead Build Up in Children 5

Solution for Model of Activity in a Small Girl The differential equation is written:

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- Make the substitution, $z(t) = a(t) - 12$, with $z'(t) = a'(t)$, so

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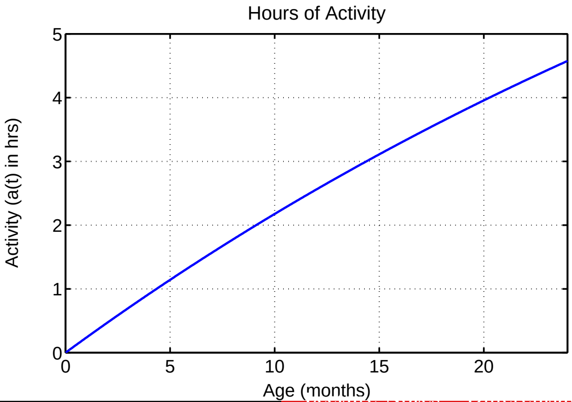
- The solution is $z(t) = -12\,e^{-0.02t} = a(t) - 12$ or
$$a(t) = 12(1 - e^{-0.02t})$$

Lead Build Up in Children

Graph of Baby Activity: Note that

$$a(24) = 12(1 - e^{-0.48}) = 4.57 \text{ hr}$$



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Lead Accumulation in the Child The lead will enter the girl's body proportional to her activity time

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- Find the solution $P(t)$ and graph this solution

Lead Build Up in Children 8

Solution: Lead Accumulation in the Child The model satisfies $P(0) = 0$ with

$$\frac{dP}{dt} = 200(12(1 - e^{-0.02t})) = 2400(1 - e^{-0.02t})$$

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- Integration is representative of the accumulation or sum of the past lead acquired through the playing activity
- Obtain the solution from

$$P(t) = 2400 \int (1 - e^{-0.02t})dt$$

Lead Build Up in Children

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Lead Build Up in Children

Solution: Lead Accumulation in the Child

$$\begin{aligned} P(t) &= 2400 \int (1 - e^{-0.02t})dt \\ &= 2400 \left(\int 1dt - \int e^{-0.02t} dt \right) \end{aligned}$$

Lead Build Up in Children

Solution: Lead Accumulation in the Child

$$
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P(t) &= 2400 \int (1 - e^{-0.02t})dt \\
&= 2400 \left(\int 1dt - \int e^{-0.02t} dt \right) \\
&= 2400 \left(t - \frac{e^{-0.02t}}{-0.02} + C \right)
\end{aligned}
$$

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&= 2400 \left(t + 50\, e^{-0.02t} + C \right)
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Lead Build Up in Children

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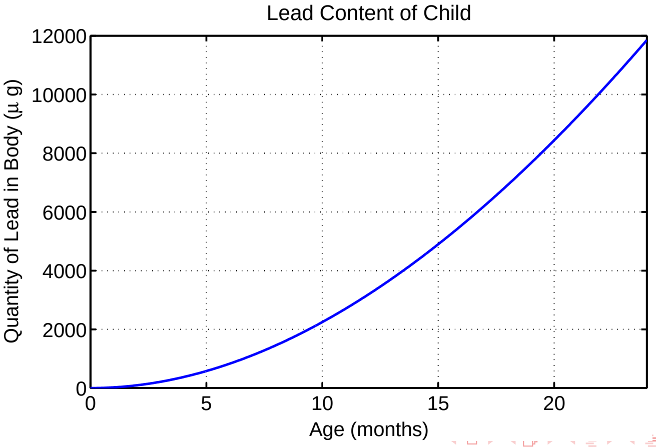
It follows that $C = -50$ and the solution is

$$P(t) = 2400\left(t + 50\,e^{-0.02t} - 50\right),$$

where $P(t)$ is measured in μg accumulated by t months

Lead Build Up in Children

Graph of Lead Build Up in a Child



Lead Build Up in Children 12

Concentration of Lead in the Child: The solution above gives the amount of lead in the girl,

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but it is the concentration that matters

- Assume that this lead is uniformly distributed throughout the body
- Find the lead level measured from blood samples at ages one and two
- Typical measurements are in terms of μg/dl
- Note that 10 dl of blood weighs about 1 kg, and a girl at age one (12 months) weighs 10 kg, while she weighs about 12 kg at age 2 (24 months)

Lead Build Up in Children 13

Solution: Concentration of Lead: From the solution $P(t)$

$$P(12) = 2400\left(12 + 50\,e^{-0.24} - 50\right) = 3195\ \mu\text{g},$$

Lead Build Up in Children

Solution: Concentration of Lead: From the solution $P(t)$

$$P(12) = 2400\left(12 + 50\,e^{-0.24} - 50\right) = 3195\ \mu\text{g},$$

- Since the girl weighs 10 kg, the concentration is given by

$$c(12) = \frac{3195\ \mu\text{g}}{10\ \text{kg}} = 31.95\ \mu\text{g/dl}$$

Lead Build Up in Children

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