December 16, 2004 Math 121a Name ______________________________

Lab Section __________________

1. (20pts) Differentiate the following functions (you **don't** have to simplify):

a. $f(x) = 7x^3 - \dfrac{5}{e^{2x}} + (x^3 - 5x)\ln(x)$.

$f'(x) =$ ____

b. $g(t) = 10\, e^{t/5} - \dfrac{t^2+5}{2-e^{-t}} + \dfrac{8}{\sqrt{t}}$.

$g'(t) =$ ____

2. (35pts) Give the derivative of the function. Find the x and y-intercepts and any asymptotes. Determine all extrema, maxima and minima (including the x and y values). Sketch the function.

a. $y = 12x - x^3$,

$y'(x) =$ ______________________________

x-intercepts are ____________________ Sketch the graph.

y-intercept is ________

$(x_{1c}, y(x_{1c})) =$ _________________

Max or Min ________

$(x_{2c}, y(x_{2c})) =$ _________________

Max or Min ________

b. $y = (x + 2)e^{-0.2x}$.

$y'(x) =$ ______________________________

x-intercept is ________ y-intercept is ________

Vertical Asymptote at $x =$ ________ Horizontal Asymptote at $y =$ ________

(Write "None" if it doesn't exist.)

$(x_{1c}, y(x_{1c})) =$ _________________ Sketch the graph.

Max or Min ________

3. (25pts) Davies and Smith (1997) (*Global Ecology and Biogeography* **7**, p. 285-294) collected data on Lepidoptera in the West Indies. They gathered a large amount of data, but we will examine only a couple pieces of information from their data set. They found that on the Biminis with an area (A) of 22 km^2 there were 20 species (N) of butterflies, while St. Lucia with an area of 617 km^2 there were 48 species of butterflies. (Give all answers to **4 significant figures**.)

a. A linear model for the number of species (N) as a function of the area (A) is given by

$$N = mA + b$$

for some constants m and b. Find the constants m and b. Use this model to predict the number of species on Grenada with an area of 345 km^2. Also, estimate the area of Puerto Rico with this model given that it has 97 species of butterflies.

$m =$ __________

$b =$ __________

If $A = 345$ km^2, then $N =$ __________ species,

If $N = 97$ species, then $A =$ __________ km^2.

b. An allometric model for the number of species (N) as a function of the area (A) is given by

$$N = kA^a$$

for some constants k and a. Find the constants k and a. Use this model to predict the number of species on Grenada with an area of 345 km^2. Also, estimate the area of Puerto Rico with this model given that it has 97 species of butterflies.

$k =$ __________

$a =$ __________

If $A = 345$ km^2, then $N =$ __________ species,

If $N = 97$ species, then $A =$ __________ km^2.

c. Which model provides the better estimate? Give a brief reason for your choice.

4. (20pts) a. A culture of bacteria satisfies the Malthusian growth equation

$$P_{n+1} = 1.014 P_n, \qquad P_0 = 4000,$$

where n is in minutes. Give the general solution (in only n and P_0) for this growth equation and state its population after 60 minutes. Determine how long it takes for this culture to double.

$P_n =$ ____________________

$P_{60} =$ __________

Doubling time = __________

b. Another culture of bacteria satisfies a similar Malthusian growth law,

$$B_{n+1} = (1 + r)B_n.$$

Suppose that this culture doubles in 40 min and starts with 1000 bacteria. Find the general solution for this culture and determine what its population is after 60 min. long until the population of this bacteria is the same as the original culture from Part a. Give both the time and population when they are equal.

$r =$ __________

$B_n =$ ____________________

$B_{60} =$ __________

Populations equal at $n_{eq} =$ __________ with $P_{n_{eq}} = B_{n_{eq}} =$ ____________________

5. (25pts) a. A normal subject is given a supply of air enriched with Helium. Assume her breathing satisfies the model,

$$c_{n+1} = (1-q)c_n + q\gamma,$$

where c_n is the concentration of Helium, q is the fraction of air exchanged, and γ, the ambient concentration of Helium. Assume that $c_0 = 200$ ppm, $q = 0.16$, and $\gamma = 5.2$ ppm. Find the expected concentrates in the next two breaths c_1 and c_2 after breathing normally in the room, then determine the equilibrium concentration in her lungs after many breaths.

$c_1 =$ ____________ ppm and $c_2 =$ ____________ ppm

Equilibrium concentration $c_e =$ ____________ ppm

b. A woman with a chronic lung problem undergoes the same test. Experimental measurements show the following concentrations of the exhaled air after she resumes normal breathing in the room.

Breath Number	0	1	2
Conc. of He (ppm)	200	176	155

There was a leak of Helium, so the concentration of Helium in the room is higher than before, but not known. It is assumed that γ is constant as is her breathing fraction, q. Use the data above to find the constants q and γ. Then determine the concentration of Helium in the next two breaths, c_3 and c_4. Find the equilibrium concentration of Helium in this subject's lungs based on this breathing model. What is the stability of this equilibrium concentration? (If a solution moves closer to an equilibrium point, then it is probably stable. If it moves away, then it is most likely unstable.)

$q =$ ____________ and $\gamma =$ ____________ ppm

$c_3 =$ ____________ ppm and $c_4 =$ ____________ ppm

Equilibrium concentration $c_e =$ ____________ ppm

Stable or Unstable ______________

6. (20pts) In lab we saw the experimental fit of O_2 consumption (in μl/hr) after a blood meal by the beetle *Triatoma phyllosoma*. Below is a cubic polynomial fit to measurements for a different individual "kissing bug,"

$$Y(t) = \frac{1}{3}t^3 - \frac{13}{2}t^2 + 30\,t + 100,$$

where t is in hours, for $0 \le t \le 12$.

a. Find the rate of change in O_2 consumption per hour, $\frac{dY}{dt}$. What is the rate of change in the O_2 consumption at $t = 4$? Also, compute $Y''(t)$. When is the rate of change in O_2 consumption per hour decreasing the most and what is that maximum rate of decline?

$Y'(t) =$ ________________________________ $Y'(4) =$ __________

$Y''(t) =$ __________________________

Rate of maximum decrease at $t_{dec} =$ __________ $Y'(t_{dec}) =$ __________

b. Use the derivative to find when the minimum and maximum O_2 consumption for this beetle occurs during the experiment. Give the O_2 consumption at those times.

$t_{max} =$ ________________ $Y(t_{max}) =$ ________________

$t_{min} =$ ________________ $Y(t_{min}) =$ ________________

7. (30pts) a. It has been shown that iron is the primary limiting nutrient in open ocean waters. There are currently a number of experiments to see if seeding the ocean with iron can create an algal bloom that fixes CO_2 (to remove this greenhouse gas). Soluable iron that is dumped into the ocean is rapidly used by algae, which are consumed by other organisms. At $t = 0$, a research vessel from Scripps Institute of Oceanography dumps 500 kg of soluable iron. Measurements from a trailing ship indicate that the amount of iron remaining in the water (not in the algae) satisfies the equation:

$$F(t) = 500\, e^{-0.23t},$$

where t is in days. Find how long it takes for the amount of soluable iron to reach the level of 100 kg remaining. Sketch a graph of F showing the F-intercept and the horizontal asymptote.

$F(t) = 100$ when $t =$ __________

$F(0) =$ __________

Horizontal Asymptote at $F =$ __________

Sketch of graph.

b. Find the derivative $\dfrac{dF}{dt}$. Determine the rate of change of soluable iron at $t = 2$.

$F'(t) =$ __________________________ $F'(2) =$ __________

c. As noted above the algae rapidly blooms, then fades as the iron passes to organisms higher in the food web. Suppose that samples of the sea water give a population of algae, $P(t)$, (in thousands/cc) satisfying the following equation:

$$P(t) = 10\left(e^{-0.05t} - e^{-0.8t}\right),$$

where t is in days. Find the derivative $\dfrac{dP}{dt}$. Find when the algal population achieves its maximum concentration and determine what its maximum concentration is. Find the P-intercept and any horizontal asymptotes.

$P'(t) =$ ____________________________________

$t_{max} =$ __________ $P(t_{max}) =$ __________

$P(0) =$ __________

Horizontal Asymptote at $P =$ __________

8. (25pts) Jacob and Monod developed the theory of genetic control by induction. This is a very important control made most famous by the *lac* operon. The enzyme β-galactosidase is induced to catalyze the break down of lactose into simple sugars (glucose and galactose) for energy in the cell. The rate of induction for β-galactosidase is given by the formula

$$R(L) = \frac{V_{max} L^2}{K + L^2},$$

where L is the concentration of lactose and V_{max} and K are kinetic constants.

a. Suppose that $V_{max} = 100$ and $K = 10$. Differentiate this rate function and also find its second derivative. Give both the L and R values for any points of inflection ($L \geq 0$).

$R'(L) =$ ______________________________

$R''(L) =$ ______________________________

Point of inflection at $L_p =$ __________ with $R(L_p) =$ __________

b. Find any intercepts and asymptotes for $R(L)$.

L-intercept is __________ R-intercept is __________

Vertical Asymptote at $L =$ __________ Horizontal Asymptote at $R =$ __________
(Write "None" if it doesn't exist.)

c. Find the maximum change in rate of induction, *i.e.*, what value of L makes $R'(L)$ have its largest value and what value is that maximum change in rate of induction?

Maximum induction at $L_{max} =$ __________ with $R'(L_{max}) =$ __________