

1. a. The solutions to the logistic growth models are

$$X(t) = \frac{X_0 M}{X_0 + (M - X_0)e^{-rt}},$$

$$Y(t) = \frac{Y_0 N}{Y_0 + (N - Y_0)e^{-st}}.$$

The best fitting parameters to the data are given by:

$$\begin{aligned} X_0 &= 0.6734 & Y_0 &= 0.6341, \\ r &= 0.10987 & s &= 0.068094, \\ M &= 9.4992 & N &= 6.4139. \end{aligned}$$

The sum of square errors for the model for species  $X$  is 0.033654. The sum of square errors for the model for species  $Y$  is 0.040219. The carrying capacities for species  $X$  and  $Y$  are 9.4992 and 6.4139, respectively.

b. There are four equilibria for the system of differential equations. One is the trivial equilibrium  $(0, 0)$ . Two are the extinction equilibria, extinction of  $Y$  at  $(9.4992, 0)$  and extinction of  $X$  at  $(0, 6.4139)$ . Finally, the coexistence equilibrium is  $(2.3587, 3.30357)$ .

c. The simulation of the system of differential equations gives the following values at the times listed.

$t$ (hr)	Species $X$	Species $Y$
25	2.789	1.295
50	5.579	1.1456
100	8.923	0.1298
200	9.498	0.0001794

This model exhibits competitive exclusion, which for the given initial conditions leads to the extinction of species  $Y$  and the eventual leveling off of species  $X$  at a population of 9.4992 ( $\times 1000/\text{cc}$ ). The maximum population of species  $Y$  occurs at  $t = 34$  hr with a population of 1.3872 ( $\times 1000/\text{cc}$ ).

2. a. The solution to the damped oscillator is given by;

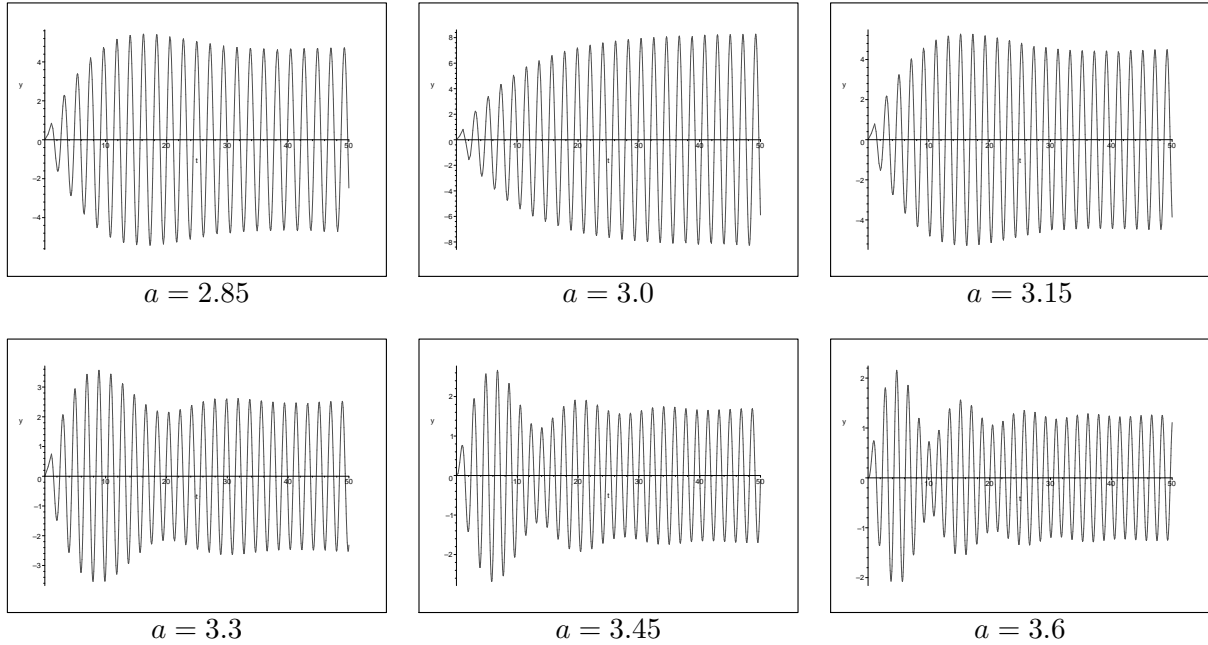
$$y(t) = \frac{11}{15} e^{-t/5} \sin(3t) + e^{-t/5} \cos(3t)$$

The first relative (and absolute) maximum occurs at  $(0.18873, 1.1915)$ , while the second (lower) relative maximum occurs at  $(2.2831, 0.78374)$ . The first relative (and absolute) minimum occurs at  $(1.2359, -0.96635)$ , while the second (lower) relative minimum occurs at  $(3.3303, -0.63565)$ .

b. We show the solution for the forced damped oscillator model with  $k = 9.01$  and  $a = 2.85$  is

$$y(t) = -3.7037 e^{-0.1t} \sin(3.0t) + 2.5617 e^{-0.1t} \cos(3.0t) + 3.9886 \sin(2.85t) - 2.5617 \cos(2.85t)$$

It follows that  $y(2) = -1.47363$  and  $y(20) = -0.74289$ . It is not hard to see that for  $a = 3.0$ , we obtain  $y(2) = -1.5206$  and  $y(20) = 6.8127$ .

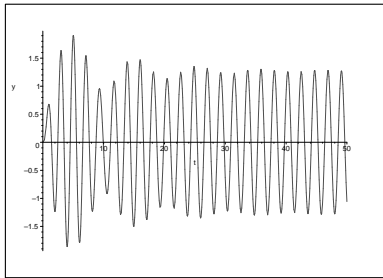


c. Below we show the plots of the 6 test sound signals and how they deflect the RLF.

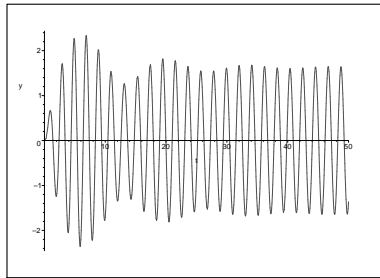
From the figures we see that for  $a = 2.85$ , the maximum response is between 5 and 5.5. For  $a = 3.0$ , the maximum response is between 8 and 8.5. For  $a = 3.15$ , the maximum response is between 5 and 5.5. For  $a = 3.3$ , the maximum response is between 3.5 and 4. For  $a = 3.45$ , the maximum response is between 2.5 and 3. For  $a = 3.6$ , the maximum response is between 2 and 2.5. The plots show that the only test sound that elicits a neural response is  $a = 3.0$  (which happens to be close to the  $\sqrt{k}$ ).

d. For  $k = 12.01$ , we show the plots of the 6 test sound signals and how they deflect the RLF.

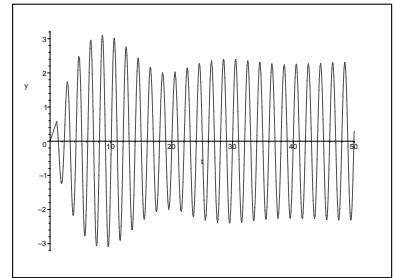
From the figures we see that for  $a = 2.85$ , the maximum response is between 1.5 and 2.0. For  $a = 3.0$ , the maximum response is between 2 and 2.5. For  $a = 3.15$ , the maximum response is between 3 and 3.5. For  $a = 3.3$ , the maximum response is about 4.5. For  $a = 3.45$ , the maximum response is between 7 and 7.5. For  $a = 3.6$ , the maximum response is between 4.5 and 5. The plots show that the only test sound that elicits a neural response is  $a = 3.45$  (which happens to be close to the  $\sqrt{k}$ ).



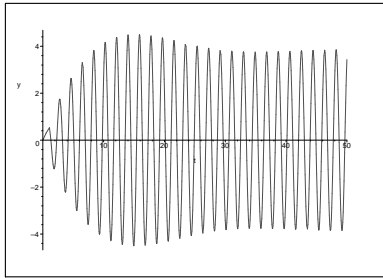
$a = 2.85$



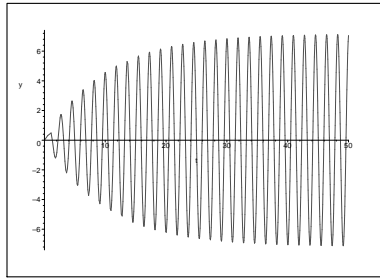
$a = 3.0$



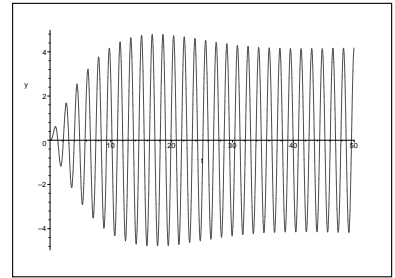
$a = 3.15$



$a = 3.3$



$a = 3.45$



$a = 3.6$