1. a. The solutions to the logistic growth models are

$$\begin{aligned} X(t) &= \frac{X_0 M}{X_0 + (M - X_0)e^{-rt}}, \\ Y(t) &= \frac{Y_0 N}{Y_0 + (N - Y_0)e^{-st}}. \end{aligned}$$

The best fitting parameters to the data are given by:

$$\begin{aligned} X_0 &= 0.6734 & Y_0 &= 0.6341, \\ r &= 0.10987 & s &= 0.068094, \\ M &= 9.4992 & N &= 6.4139. \end{aligned}$$

The sum of square errors for the model for species X is 0.033654. The sum of square errors for the model for species Y is 0.040219. The carrying capacities for species X and Y are 9.4992 and 6.4139, respectively.

b. There are four equilibria for the system of differential equations. One is the trivial equilibrium $(0, 0)$. Two are the extinction equilibria, extinction of Y at $(9.4992, 0)$ and extinction of X at $(0, 6.4139)$. Finally, the coexistence equilibrium is $(2.3587, 3.30357)$.

c. The simulation of the system of differential equations gives the following values at the times listed.

t (hr)	Species X	Species Y
25	2.789	1.295
50	5.579	1.1456
100	8.923	0.1298
200	9.498	0.0001794

This model exhibits competitive exclusion, which for the given initial conditions leads to the extinction of species Y and the eventual leveling off of species X at a population of 9.4992 ($\times 1000$/cc). The maximum population of species Y occurs at $t = 34$ hr with a population of 1.3872 ($\times 1000$/cc).

2. a. The solution to the damped oscillator is given by;

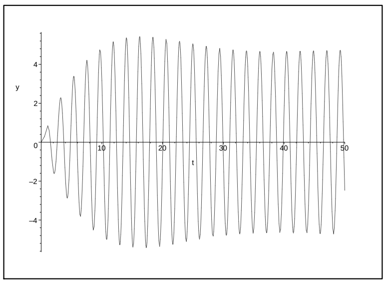
$$y(t) == \frac{11}{15} e^{-t/5} \sin(3\,t) + e^{-t/5} \cos(3\,t)$$

The first relative (and absolute) maximum occurs at $(0.18873, 1.1915)$, while the second (lower) relative maximum occurs at $(2.2831, 0.78374)$. The first relative (and absolute) minimum occurs at $(1.2359, -0.96635)$, while the second (lower) relative minimum occurs at $(3.3303, -0.63565)$.

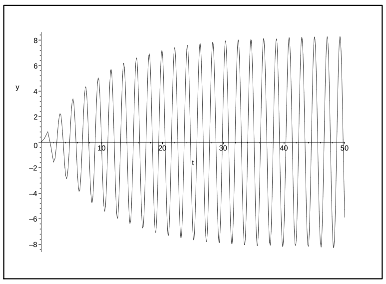
b. We show the solution for the forced damped oscillator model with $k = 9.01$ and $a = 2.85$ is

$$y(t) = -3.7037\, e^{-0.1\,t} \sin(3.0\,t) + 2.5617\, e^{-0.1\,t} \cos(3.0\,t) + 3.9886\, \sin(2.85\,t) - 2.5617\, \cos(2.85\,t)$$

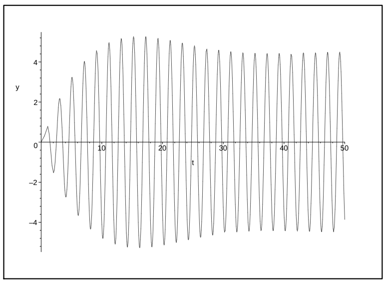
It follows that $y(2) = -1.47363$ and $y(20) = -0.74289$. It is not hard to see that for $a = 3.0$, we obtain $y(2) = -1.5206$ and $y(20) = 6.8127$.



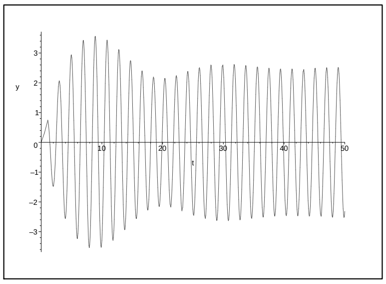
$a = 2.85$



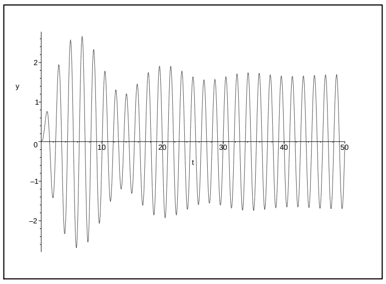
$a = 3.0$



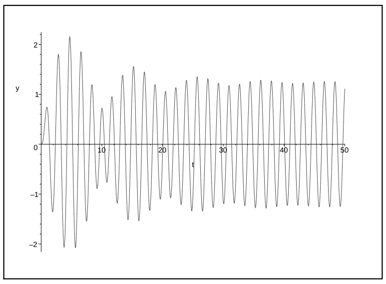
$a = 3.15$



$a = 3.3$



$a = 3.45$



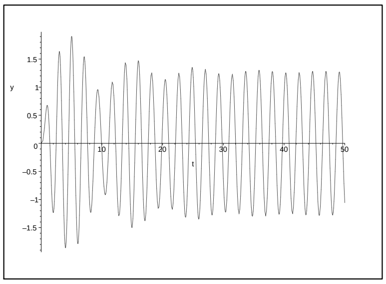
$a = 3.6$

c. Below we show the plots of the 6 test sound signals and how they deflect the RLF.

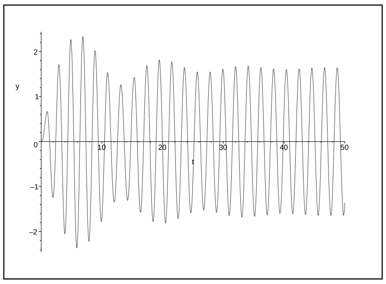
From the figures we see that for $a = 2.85$, the maximum response is between 5 and 5.5. For $a = 3.0$, the maximum response is between 8 and 8.5. For $a = 3.15$, the maximum response is between 5 and 5.5. For $a = 3.3$, the maximum response is between 3.5 and 4. For $a = 3.45$, the maximum response is between 2.5 and 3. For $a = 3.6$, the maximum response is between 2 and 2.5. The plots show that the only test sound that elicits a neural response is $a = 3.0$ (which happens to be close to the $\sqrt{k}$).

d. For $k = 12.01$, we show the plots of the 6 test sound signals and how they deflect the RLF.

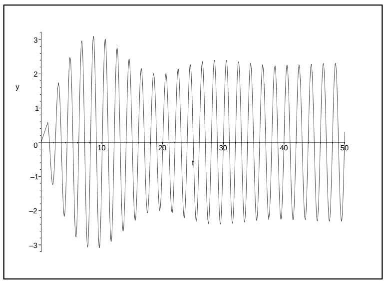
From the figures we see that for $a = 2.85$, the maximum response is between 1.5 and 2.0. For $a = 3.0$, the maximum response is between 2 and 2.5. For $a = 3.15$, the maximum response is between 3 and 3.5. For $a = 3.3$, the maximum response is about 4.5. For $a = 3.45$, the maximum response is between 7 and 7.5. For $a = 3.6$, the maximum response is between 4.5 and 5. The plots show that the only test sound that elicits a neural response is $a = 3.45$ (which happens to be close to the $\sqrt{k}$).



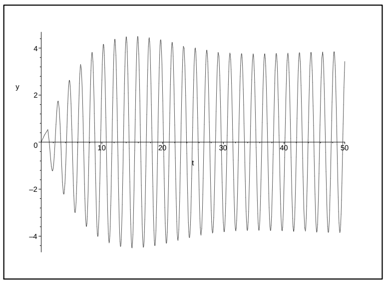
$a = 2.85$



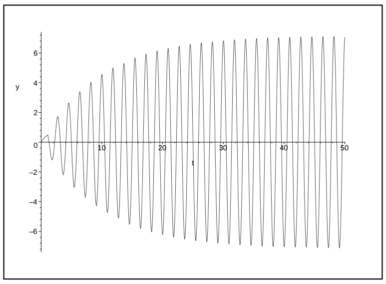
$a = 3.0$



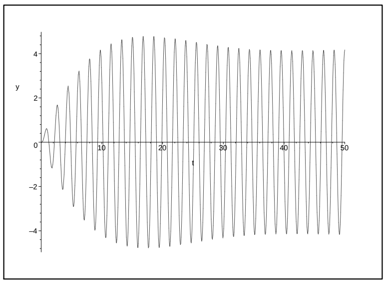
$a = 3.15$



$a = 3.3$



$a = 3.45$



$a = 3.6$