1. a. With the model given by

$$G(t) = G_0 + Ae^{-\alpha t}\cos(\omega(t-\delta)),$$

the first set of data gives the best fitting parameters $G_0 = 83.893$, $A = 175.813$, $\alpha = 0.9133$, $\omega = 1.87045$, and $\delta = 0.87294$. It follows that best model is:

$$G_1(t) = 83.893 + 175.813\, e^{-0.9133t}\cos(1.87045(t - 0.87294)).$$

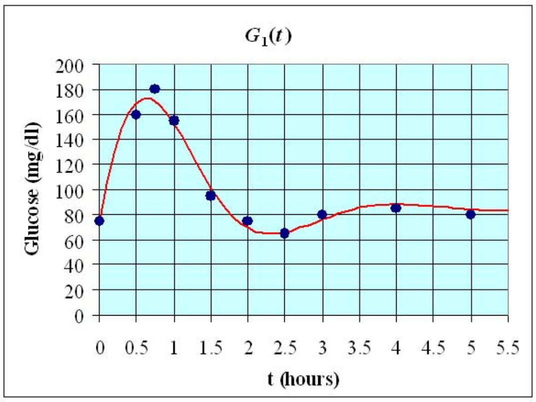
The sum of square errors is 303.108.

The second set of data gives the best fitting parameters $G_0 = 106.075$, $A = 207.729$, $\alpha = 0.4934$, $\omega = 1.1133$, and $\delta = 1.4214$. It follows that best model is:

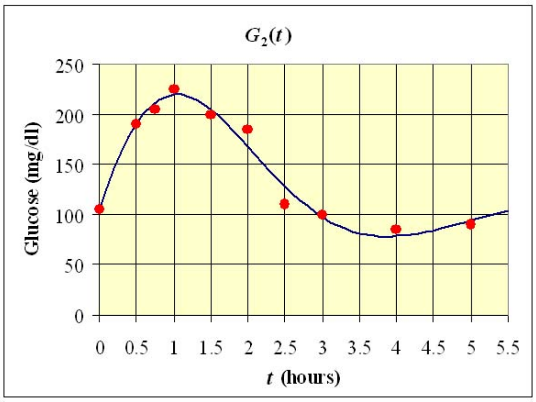
$$G_2(t) = 106.075 + 207.729\, e^{-0.4934t}\cos(1.1133(t - 1.4214)).$$

The sum of square errors is 781.406.

b. Below are graphs of the data for the models fitting these patients. The absolute maximum for $G_1(t)$ is $t_{max} = 0.63010$ hr with $G_1(t_{max}) = 172.751$ mg/dl of blood, while the absolute minimum is $t_{min} = 2.3097$ hr with $G_1(t_{min}) = 64.728$ mg/dl of blood. The absolute maximum for $G_2(t)$ is $t_{max} = 1.0467$ hr with $G_2(t_{max}) = 219.38$ mg/dl of blood, while the absolute minimum is $t_{min} = 3.8686$ hr with $G_1(t_{min}) = 77.918$ mg/dl of blood.



$G_1(t)$ and data



$G_1(t)$ and data

c. From the model for the first patient, $G_1(t)$, $\omega_0 = \sqrt{\omega^2 + \alpha^2} = 2.0815$. Since $2\pi/\omega_0 = 3.0186 < 4$, this patient is normal. From the model for the second patient, $G_2(t)$, $\omega_0 = \sqrt{\omega^2 + \alpha^2} = 1.218$. Since $2\pi/\omega_0 = 5.160 > 4$, this patient is diabetic.

2. a. The solution for the simple model for pollution in Lake Erie is given by:

$$c(t) = k + (c_0 - k)e^{-\frac{35}{92}t}.$$

b. Using the data from the table, we find that the best fitting constants are $k = 4.8695$ and $c_0 = 2.0092$ with the sum of square errors being 0.031842. It follows that the best fitting solution is given by

$$c(t) = 4.8695 - 2.8603\, e^{-\frac{35}{92}\, t}.$$

c. The solution for this problem where we let the time at Year 5 be $t = 0$ is

$$c(t) = 4.5\, e^{-\frac{35}{92}\, t}.$$

It follows that the concentration of pollutant drops to half the amount in Year 5 when $t = 1.8220$ or less than two years later. $c(5) = 0.67160$ ppm and $c(10) = 0.10023$ ppm.

d. From the Euler solution, we find the approximate solution at $t = 1$ is $c(1) \simeq 4.41358$, $t = 2$ is $c(2) \simeq 4.16112$, $t = 3$ is $c(3) \simeq 3.82444$, $t = 5$ is $c(5) \simeq 3.08262$, $t = 7$ is $c(7) \simeq 2.39577$, and $t = 10$ is $c(10) \simeq 1.58579$. It is easy to see that these values are substantially higher than the ones in Part c.

e. From Maple's **dsolve**, the solution of the modified model for loss of pollution after a ban takes place is given by:

$$c(t) = 7.4292\, e^{-0.15\, t} - 2.9292\, e^{-\frac{35}{92}\, t}.$$

This modified model takes $t = 7.4766$ years for the pollutant level to fall to half the concentration in Year 5. The solution at $t = 10$ is $c(10) = 1.5924$. The percent error between the Euler approximation in Part c and the actual solution is given by

$$100\frac{(1.58579 - 1.5924)}{1.5924} = -0.415\%.$$