1. a. The best model for the growth of the puppy has the parameters $W_0 = 1.767$, $M = 34.09$, and $r = 0.024935$, which fits the model

$$W(t) = \frac{1.767(34.09)e^{0.024935t}}{34.09 + 1.767(e^{0.024935t} - 1)} = \frac{60.237e^{0.024935t}}{32.323 + 1.767e^{0.024935t}}.$$

Since the time t is in days (1 year = 365 days), $W(365) = 34.02$ kg and $W(5 \times 365) = 34.09$ kg.

b. Excel's trendline fitting the best allometric model to the weight and calorie data gives $k = 135.52$ and $a = 0.7436$, so the model is

$$C(W) = 135.52\, W^{0.7436}.$$

c. The composite function for the amount of calories needed as a function of age (not taking into account the higher activity of puppies) is

$$C(t) = 135.52 \left(\frac{60.237e^{0.024935t}}{32.323 + 1.767e^{0.024935t}} \right)^{0.7436}.$$

With the help of Maple,

$$C'(t) = .291051729.4\, e^{0.024935\, t} \left(\frac{e^{0.024935\, t}}{32323.0 + 1767.0\, e^{0.024935\, t}} \right)^{-\frac{641}{2500}} \left(32323.0 + 1767.0\, e^{0.024935\, t}\right)^{-2}.$$

d. The point of inflection occurs at $t = 104.7$ days with a daily calorie intake of $C(104.7) = 991.9$ calories and the rate of change in calorie intake increasing at a rate of $C'(104.7) = 10.55$ calories/day.

2. a. For the logistic growth model, the best values of the parameters that fit the data are $r = 1.558$ and $m = 0.001654$, which gives the best fit model

$$P_{n+1} = 1.558P_n - 0.001654P_n^2.$$

The least sum of square errors between the data and this model is 4620.92.

For the Beverton-Holt model, the best values of the parameters that fit the data are $a = 1.7389$ and $b = 0.002165$, which gives the best fit model

$$P_{n+1} = \frac{1.7389P_n}{1 + 0.002165P_n}.$$

The least sum of square errors between the data and this model is 4637.79. One can easily see that the logistic model is only slightly better according to the sum of square errors.

b. The equilibria for the logistic model are $P_e = 0$ and $P_e = 337.36$. The derivative of this updating function is given by

$$f'(P) = 1.558 - 0.003308\, P.$$

At $P_e = 0$, $f'(0) = 1.558$, which shows that this equilibrium is unstable with solutions monotonically growing away from it. At $P_e = 337.36$, $f'(337.36) = 0.442$, which shows that this equilibrium is stable with solutions monotonically growing toward this equilibrium, the carrying capacity.

The equilibria for the Beverton-Holt model are $P_e = 0$ and $P_e = 341.29$. The derivative of this updating function is given by

$$B'(P) = \frac{1.7389}{(1 + 0.002165\,P)^2}.$$

At $P_e = 0$, $B'(0) = 1.7389$, which shows that this equilibrium is unstable with solutions monotonically growing away from it. At $P_e = 341.29$, $B'(341.29) = 0.5751$, which shows that this equilibrium is stable with solutions monotonically growing toward this equilibrium, the carrying capacity.

c. The discrete logistic growth model best fits the data with an initial population of $P_0 = 11.783$, which gives a sum of square errors of 3250.90. We find that the population at $t = 10$ weeks is $P_5 = 90.726$, the population at $t = 20$ weeks is $P_{10} = 294.61$, and the population at $t = 30$ weeks is $P_{15} = 336.40$.

The Beverton-Holt model best fits the data with an initial population of $P_0 = 7.606$, which gives a sum of square errors of 2938.36. We see that this best fitting initial condition is closer to the actual starting condition, and this model fits the time series data better according to the sum of square errors though not very much better. We find that the population at $t = 10$ weeks is $P_5 = 90.784$, the population at $t = 20$ weeks is $P_{10} = 290.82$, and the population at $t = 30$ weeks is $P_{15} = 337.61$. These simulations are very similar, not matching the data well at the beginning, but following the growth phase quite well, then reasonably matching what appears to be a carrying capacity around 340.