1. a. Rewrite the second integral as a power, then

$$\begin{aligned}\int \left(6\cos(3x) - \frac{2}{x^3}\right) dx &= 6\int \cos(3x)dx - 2\int x^{-3}dx \\ &= 6\frac{\sin(3x)}{3} - 2\frac{x^{-2}}{-2} + C = 2\sin(3x) + \frac{1}{x^2} + C\end{aligned}$$

b. These are basic integral forms

$$\begin{aligned}\int \left(4x + e^{-3x}\right) dx &= 4\int x\,dx + \int e^{-3x}dx \\ &= 4\frac{x^2}{2} + \frac{e^{-3x}}{-3} + C = 2x^2 - \frac{1}{3}e^{-3x} + C\end{aligned}$$

c. The first integral is written as a power, while the second integral uses the substitution $u = 3\,x - 2$, so $du = 3\,dx$.

$$\begin{aligned}\int \left(3\,x^{-2} + 3\cos(3\,x - 2)\right) dx &= 3\int x^{-2}dx + \int \cos(u)du \\ &= 3\frac{x^{-1}}{-1} + \sin(u) + C = -\frac{3}{x} + \sin(3\,x - 2) + C\end{aligned}$$

d. Let $u = -x^2$, so $du = -2\,x\,dx$.

$$\begin{aligned}\int \left(2x\,e^{-x^2} - 4\,x\right) dx &= -\int e^u\,du - 4\frac{x^2}{2} \\ &= -e^u - 2\,x^2 + C = -e^{-x^2} - 2\,x^2 + C\end{aligned}$$

e. Rewrite the second integral as a power, then

$$\begin{aligned}\int \left(4e^{-2x} + \frac{3}{\sqrt{x}}\right) dx &= -2e^{-2x} + 3\int x^{-1/2}dx \\ &= -2e^{-2x} + 6\sqrt{x} + C\end{aligned}$$

f. Expand the squared term, then

$$\begin{aligned}\int \left(5x^2 - 1\right)^2 dx &= \int \left(25\,x^4 - 10\,x^2 + 1\right) dx \\ &= 5\,x^5 - \frac{10}{3}\,x^3 + x + C\end{aligned}$$

g. Let $u = x^2 + 4\,x - 5$, so $du = (2\,x + 4)\,dx = 2(x + 2)\,dx$.

$$\begin{aligned}\int \left(x^2 + 4\,x - 5\right)^3 (x + 2)dx &= \frac{1}{2}\int \left(x^2 + 4\,x - 5\right)^3 2(x + 2)dx \\ &= \frac{1}{2}\int u^3 du \\ &= \frac{1}{8}u^4 + C \;=\; \frac{1}{8}(x^2 + 4\,x - 5)^4 + C\end{aligned}$$

h. Let $u = \sin(4x)$, so $du = 4\cos(4x), dx$.

$$\begin{aligned}\int \left(\frac{7}{x} + 8\sin^3(4x)\cos(4x)\right) dx &= 7\ln(x) + 2\int u^3\, du \\ &= 7\ln(x) + \frac{u^4}{2} + C \;=\; 7\ln(x) + \frac{\sin^4(4x)}{2} + C\end{aligned}$$

2. a. This is a time varying differential equation. It can be written

$$y(t) = \int \left(1 + e^{-t}\right) dt = t - e^{-t} + C.$$

The initial condition $y(0) = 3 = -1 + C$, which implies $C = 4$. Hence, the solution is $y(t) = t - e^{-t} + 4$.

b. This is a time varying differential equation. It can be written

$$y(t) = \int \left(2 - \frac{4}{t}\right) dt = 2\,t - 4\,\ln(t) + C.$$

The initial condition $y(1) = 5 = 2 + C$, which implies $C = 3$. Hence, the solution is $y(t) = 2\,t - 4\,\ln(t) + 3$.

c. This is a separable differential equation. It can be written

$$\int 2y\,dy = \int 3t^2\,dt \qquad \text{or} \qquad y^2 = t^3 + C.$$

It follows that $y(t) = \pm\sqrt{t^3 + C}$. The initial condition $y(0) = 4 = \sqrt{C}$, which implies $C = 16$. Hence, the solution is

$$y(t) = \sqrt{t^3 + 16}.$$

d. This is a linear differential equation, which can be written

$$\frac{dy}{dt} = -0.02(y - 100).$$

With the substitution $z(t) = y(t) - 100$, we have

$$\frac{dz}{dt} = -0.02z, \qquad z(0) = y(0) - 100 = -95.$$

Thus, $z(t) = -95\,e^{-0.02\,t}$. It follows that

$$y(t) = 100 - 95\,e^{-0.02\,t}.$$

e. This is a separable differential equation. It can be written

$$\int \frac{dy}{y} = \int \frac{2\,t\,dt}{t^2+1}.$$

The right integral uses the substitution $u = t^2 + 1$, so $du = 2\,t\,dt$. Hence,

$$\begin{aligned} \ln|y(t)| &= \int \frac{du}{u} = \ln|u| + C = \ln(t^2+1) + C \\ y(t) &= e^{\ln(t^2+1)+C} = A(t^2+1), \end{aligned}$$

where $A = e^C$. The initial condition $y(0) = 3 = A$, which implies $A = 3$. Hence, the solution is

$$y(t) = 3(t^2+1).$$

f. This is a separable differential equation. It can be written

$$\int \frac{dy}{y} = \int (2 - 0.2t)dt \qquad \text{or} \qquad \ln|y| = 2\,t - 0.1\,t^2 + C.$$

It follows that $y(t) = e^{2\,t-0.1\,t^2+C} = A\,e^{2\,t-0.1\,t^2}$ with $A = e^C$. The initial condition $y(0) = 10 = A$, which implies $A = 10$. Hence, the solution is

$$y(t) = 10\,e^{2\,t-0.1\,t^2}.$$

g. This is a time-varying differential equation, so we integrate giving

$$y(t) = \int (4 - 2\,\sin(2(t-3)))dt = 4\,t - 2\int \sin(2(t-3))dt.$$

With the substitution $u = 2(t-3)$ and $du = 2\,dt$, we have

$$y(t) = 4\,t - \int \sin(u)du = 4\,t + \cos(u) + C = 4\,t + \cos(2(t-3)) + C.$$

With the initial condition $y(3) = 5$, $12 + \cos(0) + C = 5$ or $C = -8$. It follows that

$$y(t) = 4\,t + \cos(2(t-3)) - 8.$$

h. This is a separable differential equation. It can be written

$$\int e^y dy = \int e^t dt \qquad \text{or} \qquad e^y = e^t + C.$$

It follows that $y(t) = \ln(e^t + C)$. The initial condition $y(0) = 6 = \ln(1 + C)$, which implies $C = e^6 - 1$. Hence, the solution is

$$y(t) = \ln(e^t + e^6 - 1).$$

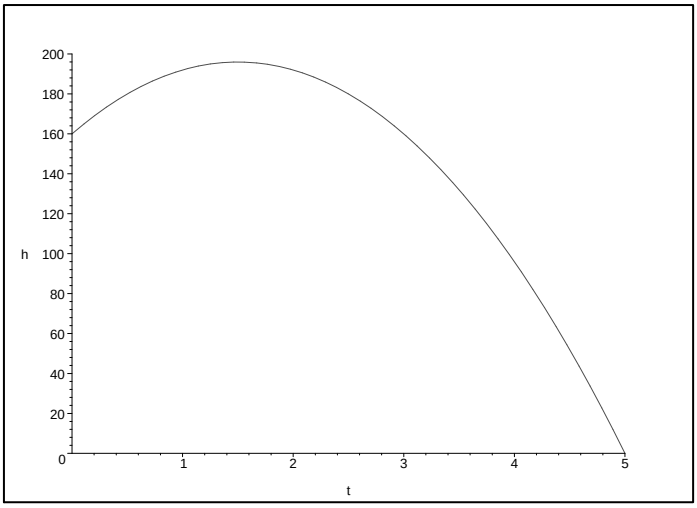
3. a. Since the acceleration of gravity is -32 ft/sec^2, the velocity of the ball is the integral, giving $v(t) = -32t + C$, which when combined with the initial condition $v(0) = 48$, gives $v(t) = 48 - 32\,t$. The velocity is integrated to give the height of the ball

$$h(t) = \int v(t)\,dt = \int (-32\,t + 48)dt = -16\,t^2 + 48\,t + C.$$

With the initial height, $h(0) = 160$, so $h(t) = -16\,t^2 + 48\,t + 160$. The maximum occurs when $v(t) = 0$, so $t = 3/2$ sec. It follows that the maximum height of the ball is $h(3/2) = 196$ ft.

b. The ball hits the ground when $h(t) = -16(t^2 - 3t - 10) = -16(t + 2)(t - 5) = 0$, so at $t = 5$ sec. The velocity is $v(5) = -112$ ft/sec.

c. The graph for the height of the ball is shown below for $t \geq 0$.



4. Integrating the acceleration due to gravity as in the previous problem, we see that the velocity is given by $v(t) = v_0 - 32\,t$. Similarly, the height is the integral of the velocity (as above), so $h(t) = \int (v_0 - 32\,t)dt = -16\,t^2 + v_0 t$, where the integration constant is zero, since the initial height is zero. The maximum height occurs when the velocity is zero, so $t = v_0/32$. But

$$h(v_0/32) = \frac{v_0^2}{32} - \frac{v_0^2}{64} = \frac{v_0^2}{64} = 8.$$

It follows that $v_0^2 = 512$ or $v_0 = 16\sqrt{2}$, which is the initial upward velocity. The length of time that the kangeroo stays in the air is twice the length of time to reach the maximum, so it stays in the air for $t = \sqrt{2}$ sec.

5. The differential equation is separable, so write

$$\int T^{-\frac{1}{2}} dT = k \int dt \qquad \text{or} \qquad 2\,T^{\frac{1}{2}}(t) = kt + C.$$

It follows that

$$T(t) = \left(\frac{kt + C}{2}\right)^2.$$

The initial condition $T(0) = 1$ implies $C = 2$, so $T(t) = \left(\frac{kt}{2} + 1\right)^2$. Since $T(4) = \left(\frac{4\,k}{2} + 1\right)^2 = 25$, $2\,k + 1 = 5$ or $k = 2$. Thus, the solution for the spread of the disease in this orchard is

$$T(t) = (t + 1)^2.$$

When $t = 10$, $T(10) = 121$.

6. a. The differential equation for Gompertz law of growth is separable. The solution can be found as follows, where we solve the complicated integral with the substitution of $u = \ln\left(\frac{N}{2000}\right)$, so $du = \frac{dN}{N}$.

$$\begin{aligned}
\frac{dN}{dt} &= -0.1N \ln\left(\frac{N}{2000}\right) \\
\int \left(\ln\left(\frac{N}{2000}\right)\right)^{-1} \frac{dN}{N} &= -0.1 \int dt = -0.1\,t + C \\
\int u^{-1} du = \ln|u| &= \ln\left(\ln\left(\frac{N(t)}{2000}\right)\right) = -0.1\,t + C \\
\ln\left(\frac{N(t)}{2000}\right) &= e^{-0.1\,t + C} = Ae^{-0.1\,t} \\
\frac{N(t)}{2000} &= e^{Ae^{-0.1\,t}} \\
N(t) &= 2000\,e^{Ae^{-0.1\,t}}
\end{aligned}$$

The initial condition is $N(0) = 10$, so $10 = 2000e^A$ or $A = -\ln(200)$. Thus, the solution is given by

$$N(t) = 2000\,e^{-\ln(200)e^{-0.1\,t}}.$$

b. For large time, $e^{-0.1\,t} \to 0$, so $N(t) \to 2000e^0 = 2000$ (in thousands). Hence, $N(t) \to 2000$ is a carrying capacity, which means that the tumor levels off with a population of 2 million cells.

7. a. The solution of the Mathusian growth model is $B(t) = 1000\,e^{0.01\,t}$. The population doubles when the bacteria reaches 2000, so $1000\,e^{0.01\,t} = 2000$ or $e^{0.01\,t} = 2$. Thus, $0.01\,t = \ln(2)$ or $t = 100\ln(2) \simeq 69.3$ min for the population to double.

b. The model with time-varying growth is a separable differential equation, so

$$\begin{aligned}
\frac{dB}{dt} &= 0.01(1 - e^{-t})\,B \\
\int \frac{dB}{B} &= 0.01 \int (1 - e^{-t}) dt \\
\ln|B(t)| &= 0.01(t + e^{-t}) + C \\
B(t) &= A\,e^{0.01(t + e^{-t})},
\end{aligned}$$

where $A = e^C$. With the initial condition, $B(0) = 1000 = A\,e^{0.01}$ or $A = 1000\,e^{-0.01}$. Thus, the solution to this time-varying growth model is

$$B(t) = 1000\,e^{0.01(t+e^{-t}-1)}.$$

c. The Malthusian growth model gives $B(5) = 1051$ and $B(60) = 1822$, while the modified growth model gives $B(5) = 1041$ and $B(60) = 1804$.

8. a. The solution to the Malthusian growth model is given by $P(t) = 100\,e^{0.2\,t}$. This population doubles when $100\,e^{0.2\,t} = 200$ or $e^{0.2\,t} = 2$, so $t = 5\ln(2) \simeq 3.466$ yrs.

b. This model, including the modification for habitat encroachment, is a separable differential equation. It can be written

$$\int \frac{dP}{P} = \int (0.2 - 0.02t)dt \qquad \text{or} \qquad \ln|P| = 0.2\,t - 0.01\,t^2 + C.$$

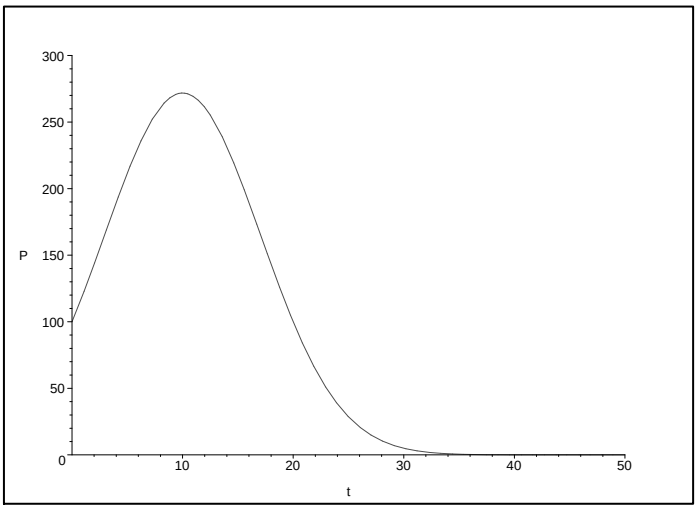
It follows that $P(t) = e^{0.2\,t-0.01\,t^2+C} = Ae^{0.2\,t-0.01\,t^2}$. The initial condition $P(0) = 100 = A$, which implies $A = 100$. Hence, the solution satisfies

$$P(t) = 100\,e^{0.2\,t-0.01\,t^2}.$$

c. We find the maximum by differentiating and setting it equal to zero,

$$P'(t) = 100\,e^{0.2\,t-0.01\,t^2}(0.2 - 0.02t) = 0.$$

So $0.2 - 0.02t = 0$, which implies that $t = 10$. Thus, the maximum of population is $P(10) = 100\,e \simeq 271.8$. If we solve $P(t) = 100\,e^{0.2\,t-0.01\,t^2} = 100$, then this is equivalent to $e^{0.2\,t-0.01\,t^2} = 1$ or $0.2\,t - 0.01\,t^2 = -0.01\,t(t-20) = 0$. Thus, either $t = 20$ (or 0), so the population returns to 100 after 20 years. The graph of the population can be seen below.



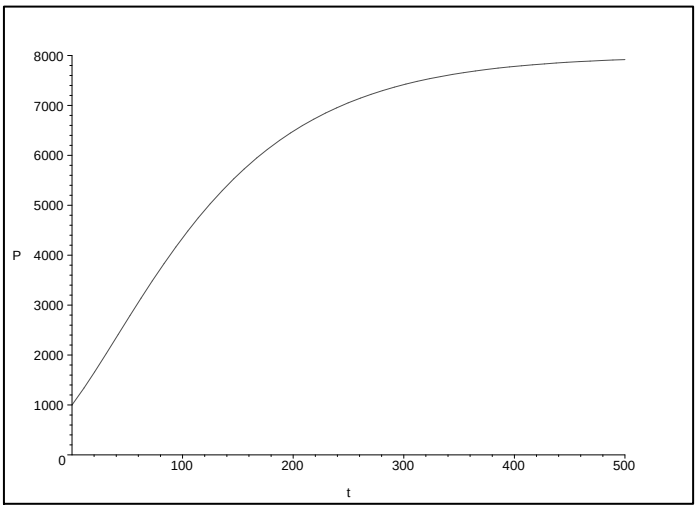
9. a. This population of cells in a declining medium satisfies a separable differential equation, which can be written

$$\int P^{-2/3}dP = \int 0.3\,e^{-0.01t}dt \qquad \text{or} \qquad 3\,P^{1/3}(t) = -30\,e^{-0.01t} + 3C.$$

It follows that $P^{1/3}(t) = -10\,e^{-0.01t} + C$, so $P(t) = \left(C - 10\,e^{-0.01t}\right)^3$. The initial condition $P(0) = 1000 = (C-10)^3$, which implies $C = 20$. The solution is given by

$$P(t) = \left(20 - 10e^{-0.01\,t}\right)^3.$$

b. This population doubles when $P(t) = \left(20 - 10e^{-0.01\,t}\right)^3 = 2000$, so $20 - 10e^{-0.01\,t} = 10\sqrt[3]{2}$ or $e^{-0.01\,t} = 2 - \sqrt[3]{2}$. It follows that $t = 100 \ln\left(\frac{1}{2-\sqrt[3]{2}}\right) \simeq 30.1$ hr. For large t, $\lim_{t\to\infty} e^{-0.01\,t} = 0$, so $\lim_{t\to\infty} P(t) = 20^3 = 8000$. Thus, there is a horizontal asymptote at $P = 8000$, so the population tends towards this value. The graph of the population can be seen below.



10. a. The change in amount of phosphate, $P(t)$, is found by adding the amount entering and subtracting the amount leaving.

$$\frac{dP}{dt} = 200 \cdot 10 - 200 \cdot c(t),$$

where $c(t)$ is the concentration in the lake with $c(t) = P(t)/10,000$. By dividing the equation by the volume, the concentration equation is given by

$$\frac{dc}{dt} = 0.2 - 0.02c = -0.02(c-10), \qquad c(0) = 0.$$

With the substitution $z(t) = c(t) - 10$, the equation above reduces to the problem

$$\frac{dz}{dt} = -0.02z, \qquad z(0) = -10,$$

which has the solution $z(t) = -10\,e^{-0.02\,t}$. Thus, the concentration is given by

$$c(t) = 10 - 10\,e^{-0.02\,t}.$$

b. The differential equation describing the growth of the algae is given by

$$\frac{dA}{dt} = 0.5(1 - e^{-0.02\,t})A^{2/3}.$$

By separating variables, we see

$$\begin{aligned}\int A^{-2/3} dA &= 0.5 \int (1 - e^{-0.02\,t}) dt \\ 3\,A^{1/3}(t) &= 0.5(t + 50\,e^{-0.02\,t}) + C \\ A(t) &= \left(\frac{0.5(t + 50\,e^{-0.02\,t}) + C}{3} \right)^3\end{aligned}$$

From the initial condition $A(0) = 1000$, we have $1000 = \left(\frac{25+C}{3}\right)^3$. It follows that $C = 5$, so

$$A(t) = \left(\frac{t + 50\,e^{-0.02\,t} + 10}{6} \right)^3.$$

11. a. The equation for the weight of the swordfish is a linear differential equation, so we first write $\dfrac{dw}{dt} = 0.015(1000 - w) = -0.015(w - 1000)$. We make the substitution $z(t) = w(t) - 1000$, giving the differential equation $\dfrac{dz}{dt} = -0.015z$ with the initial condition $z(0) = w(0) - 1000 = -1000$. Thus, $z(t) = -1000\,e^{-0.015\,t}$. It follows that $w(t) = 1000 - 1000\,e^{-0.015\,t}$. The swordfish reaches 70 kg when $1000 - 1000\,e^{-0.015\,t} = 70$ or $e^{0.015\,t} = \frac{1000}{930}$. Thus, it takes $t = \frac{200}{3} \ln\left(\frac{100}{93}\right) \simeq 4.838$ yrs to reach maturity.

b. The mercury (Hg) accumulates in swordfish according to the differential equation, which is a time varying equation. It follows that upon integration that

$$\begin{aligned}H(t) &= 0.01 \int (1000 - 1000\,e^{-0.015\,t}) dt \\ &= 10t + \frac{2000}{3}\,e^{-0.015\,t} + C\end{aligned}$$

With the initial condition $H(0) = 0$, the solution becomes

$$H(t) = 10t + \frac{2000}{3}\,e^{-0.015\,t} - \frac{2000}{3}.$$

From this equation, it follows that $H(3) = 0.665$ and $H(20) = 27.2$ mg of Hg.

c. The formula for the concentration of Hg, $c(t)$ (in μg/g) in swordfish satisfies

$$c(t) = H(t)/w(t) = \frac{10t + \frac{2000}{3}\,e^{-0.015\,t} - \frac{2000}{3}}{1000 - 1000\,e^{-0.015\,t}}.$$

It follows that $c(3) = 0.0151$ and $c(20) = 0.105$ μg/g.

12. a. Write the differential equation $\dfrac{dw}{dt} = -0.2(w - 80)$, then $z(t) = w(t) - 80$. It follows that

$$\frac{dz}{dt} = -0.2z, \qquad z(0) = -80,$$

with the solution $z(t) = -80e^{-0.2t} = w(t) - 80$. Thus,

$$w(t) = 80\left(1 - e^{-0.2t}\right).$$

For a 40 kg alligator, $w(t) = 40 = 80\left(1 - e^{-0.2t}\right)$ or $40 = 80e^{-0.2t}$, so $e^{0.2t} = 2$ or $0.2t = \ln(2)$. Thus, $t = 5\ln(2) \simeq 3.47$ years.

b. The pesticide accumulation is given by

$$\frac{dP}{dt} = 600\left(80\left(1 - e^{-0.2t}\right)\right), \qquad P(0) = 0.$$

The solution is given by

$$P(t) = 48,000\int\left(1 - e^{-0.2t}\right)dt = 48,000\left(t + 5e^{-0.2t}\right) + C.$$

The initial condition gives $P(0) = 0 = 240,000 + C$, so $C = -240,000$. Hence,

$$P(t) = 48,000\left(t + 5e^{-0.2t}\right) - 240,000.$$

The amount of pesticide in the alligator at age 5 is $P(5) = 48,000\left(5 + 5e^{-1}\right) - 240,000 = 240,000e^{-1} \simeq 88291\ \mu$g.

c. The pesticide concentration for a 5 year old alligator is

$$c(5) = \frac{P(5)}{1000w(5)} = \frac{88,291}{80,000\left(1 - e^{-1}\right)} \simeq 1.75 \text{ ppm}.$$

13. a. The differential equation can be written:

$$\frac{dc}{dt} = -0.004(c - 15),$$

so we make the substitution $z(t) = c(t) - 15$. Since $c(0) = 0$, it follows that $z(0) = -15$. The solution of the substituted equation is given by:

$$\begin{aligned} z(t) &= -15e^{-0.004t} = c(t) - 15 \\ c(t) &= 15 - 15e^{-0.004t}. \end{aligned}$$

The limiting concentration satisfies:

$$\lim_{t\to\infty} c(t) = 15 \text{ mg/m}^3.$$

b. We begin by separating variables, which gives:

$$\begin{aligned} \int \frac{dc}{c - 15} &= -0.001\int\left(4 - \cos(0.0172t)\right)dt \\ \ln(c(t) - 15) &= -0.001\left(4t - \frac{\sin(0.0172t)}{0.0172}\right) + C \\ c(t) &= 15 + Ae^{-0.001\left(4t - \frac{\sin(0.0172t)}{0.0172}\right)} \end{aligned}$$

It is easy to see that the initial condition $c(0) = 0$ implies that $A = -15$. Thus, the solution to this problem is given by:

$$c(t) = 15 - 15\, e^{-0.001\left(4t - \frac{\sin(0.0172t)}{0.0172}\right)}$$

14. a. Substituting the parameters into the differential equation gives

$$\frac{dc(t)}{dt} = \frac{200}{10000}(10 - c) = -0.02(c - 10).$$

We make the substitution $z(t) = c(t) - 10$, which gives the initial value problem $z' = -0.02z$ with $z(0) = c(0) - 10 = -10$. The solution of this differential equation is $z(t) = -10e^{-0.02t} = c(t) - 10$, so

$$c(t) = 10 - 10e^{-0.02t}.$$

We solve the equation $c(t) = 10 - 10e^{-0.02t} = 2$, so $e^{0.02t} = 10/8$ or $t = 50\ln(10/8) = 11.2$ days.

b. The Euler's formula with the linearly increasing pollutant level is given by

$$c_{n+1} = c_n + h(0.02(10 + 0.1t_n - c_n)) = c_n + 0.2 + 0.002t_n - 0.02c_n,$$

with $h = 1$. Iterating this, we create a table

$t_0 = 0$	$c_0 = 0$
$t_1 = 1$	$c_1 = c_0 + 0.2 + 0.002t_0 - 0.02c_0 = 0.2$
$t_2 = 2$	$c_2 = c_1 + 0.2 + 0.002t_1 - 0.02c_1 = 0.398$

The approximate solution is $c_2 = 0.398$ ppb.

15. a. We separate variables, so

$$\begin{aligned} \int M^{-3/4} dM &= -k \int dt \\ 4M^{1/4} &= -kt + 4C \\ M(t) &= \left(C - \frac{k}{4}t\right)^4 \end{aligned}$$

From the initial condition, $M(0) = 16 = C^4$, it follows that $C = 2$. From the information that $M(10) = 1 = (2 - 10k/4)^4$, we have $k = 0.4$. The fruit vanishes in 20 days.

b. We separate variables again to find:

$$\begin{aligned} \int M^{-3/4} dM &= -0.8 \int e^{-0.02t} dt \\ 4M^{1/4} &= \frac{0.8}{0.02} e^{-0.02t} + 4C \\ M(t) &= \left(10e^{-0.02t} + C\right)^4 \end{aligned}$$

From the initial condition, $M(0) = 16 = (10 + C)^4$, it follows that $C = -8$. Solving $10e^{-0.02t} = 8$, which is when the fruit vanishes, we find $t = 50\ln(5/4)$. Thus, the fruit vanishes in 11.157 days.

16. a. The general solution to the Malthusian growth problem with the initial condition $P(0) = 60$ is

$$P(t) = 60\, e^{rt}.$$

We are given that 2 weeks later $P(2) = 80 = 60\, e^{2r}$, so it follows that $r = \frac{1}{2}\ln\left(\frac{4}{3}\right) = 0.14384$. This gives the solution:

$$P(t) = 60\, e^{0.14384\, t}.$$

It is easy to see that the population doubles when $120 = 60\, e^{0.14384\, t}$, so $0.14384\, t_d = \ln(2)$ or the doubling time is

$$t_d = \frac{\ln(2)}{r} = 4.819 \text{ weeks}.$$

b. We begin by separating variables, so the general solution satisfies:

$$\begin{aligned} \int \frac{dP}{P} &= \int (a - b\, t)\, dt \\ \ln(P(t)) &= a\, t - \frac{bt^2}{2} + C \\ P(t) &= e^C e^{a\, t - \frac{bt^2}{2}}. \end{aligned}$$

Since the initial value is $P(0) = 60$, it follows that $e^C = 60$. Thus,

$$P(t) = 60\, e^{a\, t - \frac{bt^2}{2}}.$$

We now use the data at $t = 2$ and 4 weeks. It follows from the solution above that

$$\begin{aligned} 80 &= 60\, e^{2\, a - 2\, b} \\ 90 &= 60\, e^{4\, a - 8\, b}. \end{aligned}$$

We rearrange the terms and take logarithms of both sides to get

$$\begin{aligned} 2\, a - 2\, b &= \ln\left(\frac{4}{3}\right) \\ 4\, a - 8\, b &= \ln\left(\frac{3}{2}\right). \end{aligned}$$

We solve these equations simultaneously to obtain

$$2\, b = \ln\left(\frac{4}{3}\right) - \frac{1}{2}\ln\left(\frac{3}{2}\right),$$

so $b = 0.042475$. But $a = b + \frac{1}{2}\ln(4/3)$ or $a = 0.1863$. It follows that the solution is

$$P(t) = 60\, e^{0.1863\, t - 0.021237\, t^2}.$$

The population reaches a maximum when the derivative is zero, which occurs when $t_{max} = \frac{a}{b} = 4.3865$, so the maximum population is $P(t_{max}) = 90.286$.

17. a. With $n = 2$ and $x \in [0, 2]$, the midpoints of the interval are $x_1 = \frac{1}{2}$ and $x_2 = \frac{3}{2}$ and $\Delta x = 1$. Thus, the midpoint rule gives

$$\int_0^2 (2x - x^2)dx \simeq \left(\left(2\frac{1}{2} - \left(\frac{1}{2}\right)^2\right) + \left(2\frac{3}{2} - \left(\frac{3}{2}\right)^2\right)\right) \cdot 1 = 1.5.$$

With $n = 2$, the trapezoid rule gives

$$\int_0^2 (2x - x^2)dx \simeq \left(\frac{1}{2}(0) + (2 - 1) + \frac{1}{2}(0)\right) \cdot 1 = 1.$$

b. With $n = 4$ and $x \in [0, 2]$, the midpoints of the interval are $x_1 = \frac{1}{4}$, $x_2 = \frac{3}{4}$, $x_3 = \frac{5}{4}$, and $x_4 = \frac{7}{4}$ and $\Delta x = \frac{1}{2}$. Thus, the midpoint rule gives

$$\int_0^2 (2x - x^2)dx \simeq \left(\left(2\frac{1}{4} - \left(\frac{1}{4}\right)^2\right) + \left(2\frac{3}{4} - \left(\frac{3}{4}\right)^2\right) + \left(2\frac{5}{4} - \left(\frac{5}{4}\right)^2\right) + \left(2\frac{7}{4} - \left(\frac{7}{4}\right)^2\right)\right) \cdot \frac{1}{2} = 1.375.$$

With $n = 4$, the trapezoid rule gives

$$\int_0^2 (2x - x^2)dx \simeq \left(\frac{1}{2}0 + \left(1 - \frac{1}{4}\right) + (2 - 1) + \left(3 - \frac{9}{4}\right) + \frac{1}{2}(0)\right) \cdot \frac{1}{2} = 1.25$$

c. For $n = 2$, the midpoint rule has a 12.5% error, which is a high estimate. The trapezoid rule has a -25% error, which is a low estimate. For $n = 4$, the midpoint rule has a 3.125% error, which is a high estimate. The trapezoid rule has a -6.25% error, which is a low estimate.

18. a. The trapezoid rule applied to the data gives an approximation of the integral:

$$\int_0^{10} A(t)dt \approx \tfrac{1}{2}(0) + 1.81 + 1.62 + 1.09 + 0.66 + 0.37 + 0.2 + 0.11 + 0.05 + 0.03\tfrac{1}{2}(0.01) = 5.945$$

b. The midpoint rule gives the approximation:

$$\begin{aligned}
\int_0^{10} 4t\, e^{0.8t}dt &\approx \textstyle\sum_{i=0}^{9} 4\left(i + \frac{1}{2}\right) e^{-0.8\left(i+\frac{1}{2}\right)} \\
&= 4\left(\frac{1}{2}\right) e^{-0.8\left(\frac{1}{2}\right)} + 4\left(\frac{3}{2}\right) e^{-0.8\left(\frac{3}{2}\right)} + ... + 4\left(\frac{19}{2}\right) e^{-0.8\left(\frac{19}{2}\right)} \\
&= 1.3406 + 1.8072 + 1.3534 + 0.8513 + 0.4918 + 0.2701 + 0.1434306750 + 0.0744 \\
&\quad +0.0379 + 0.0190 \;=\; 6.3891
\end{aligned}$$

The trapezoid rule gives the approximation:

$$\begin{aligned}
\int_0^{10} 4t\, e^{0.8t}dt &\approx \tfrac{1}{2}A(0) + \textstyle\sum_{i=1}^{9} 4ie^{-0.8i} + \frac{1}{2}A(10) \\
&= 0 + 4\, e^{-0.8} + 4(2)\, e^{-0.8(2)} + ... + 2(10)e^{-0.8(10)} \\
&= 1.7973 + 1.6152 + 1.0886 + 0.6522 + 0.3663 + 0.1975 + 0.1035 + 0.0532 \\
&\quad +0.0269 + 0.0067 \;=\; 5.9074
\end{aligned}$$

c. Since the exact value of the definite integral is 6.231, then for Part a the value is low and the percent error is

$$\text{Percent Error} = 100\frac{(5.945 - 6.231)}{6.231} = -4.59\%.$$

For Part b midpoint rule, the value is high and the percent error is

$$\text{Percent Error} = 100\frac{(5.945 - 6.231)}{6.231} = 2.54\%.$$

For Part b trapezoid rule, the value is low and the percent error is

$$\text{Percent Error} = 100\frac{(5.945 - 6.231)}{6.231} = -5.19\%.$$