1. a. $f'(x) = 3\cos(3x-5) - \dfrac{3\sin(3x)}{\cos(3x)}$.

b. Rewrite the function as $g(x) = 4\left(\cos(x^2+2)\right)^{-1} - (x^2 - \sin^3(x^2))^4$, then the chain rule gives

$$g'(x) = 8x\left(\cos(x^2+2)\right)^{-2}\sin(x^2+2) - 4(x^2-\sin^3(x^2))^3(2x - 6x\sin^2(x^2)\cos(x^2)).$$

c. Use the quotient rule and product rule

$$h'(x) = \frac{(x^3+\cos(4x))(4x^3-2e^{-2x}) - (x^4+e^{-2x})(3x^2-4\sin(4x))}{(x^3+\cos(4x))^2} - e^{-x}\cos(2x) - 2e^{-x}\sin(2x).$$

d. With the product rule and chain rule,

$$k'(x) = -3x^2(x^2-5)^3\sin(x^3) + 6x(x^2-5)^2\cos(x^3) - 2\cos(2x)e^{\sin(2x)}.$$

2. a. The solution to $\dfrac{dy}{dt} = -0.2y,\ y(0) = 8$ is $y(t) = 8e^{-0.2t}$ by simple recognition.

b. The differential equation is given by $\dfrac{dx}{dt} = 3 - 0.1x = -0.1(x-30)$. We make the substitution $z(t) = x(t) - 30$ or $z(0) = 4 - 30 = -26$, since $x(0) = 4$. The modified differential equation is $z' = -0.1z$, which has the solution $z(t) = -26e^{-0.1t} = x(t) - 30$. It follows that $x(t) = 30 - 26e^{-0.1t}$.

c. The differential equation is given by $\dfrac{dw}{dt} = 0.02w + 4 = 0.02(w+200)$. We make the substitution $z(t) = w(t) + 200$ or $z(0) = 2 + 200 = 202$, since $w(0) = 2$. The modified differential equation is $z' = 0.02z$, which has the solution $z(t) = 202e^{0.02t} = w(t) + 200$. It follows that $w(t) = 202e^{0.02t} - 200$.

d. The solution to $\dfrac{dh}{dx} = -\dfrac{h}{5}$, $\quad h(0) = 50$ is $h(x) = 50e^{-x/5}$ by simple recognition.

e. This is a linear differential equation, so we first write $\dfrac{dy}{dt} = 2 + \dfrac{y}{3} = \dfrac{1}{3}(y+6)$. Thus, we make the substitution $z(t) = y(t) + 6$, giving the differential equation $\dfrac{dz}{dt} = \dfrac{1}{3}z$ with the initial condition $z(0) = y(0) + 6 = 8$. Thus, $z(t) = 8\,e^{t/3}$. It follows that $y(t) = 8\,e^{t/3} - 6$.

f. The general solution to $\dfrac{dz}{dt} = 0.3z$ is $z(t) = ce^{0.3t}$. The initial condition $z(4) = 10$ gives $z(4) = ce^{0.3(4)} = 10$, so $c = 10e^{-0.3(4)}$. It follows that $z(t) = 10e^{0.3(t-4)}$.

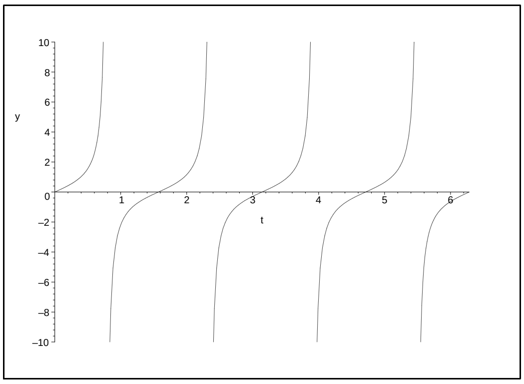
3. a. The derivative is given by

$$f'(t) = \frac{2\cos(2t)\cos(2t) + 2\sin(2t)\sin(2t)}{\cos^2(2t)} = \frac{2}{\cos^2(2t)},$$

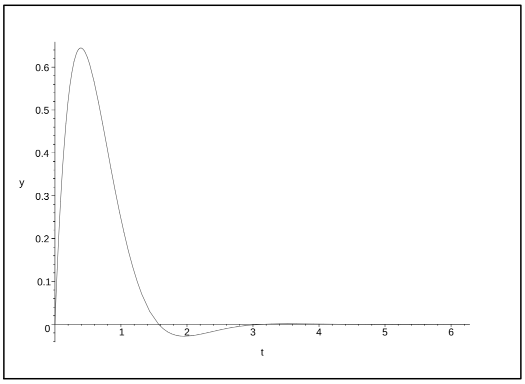
since $\sin^2(2t)+\cos^2(2t) = 1$. It follows that $f'(0) = 2\cos^2(0) = 2$. Notice that since the denominator is squared, it follows that the derivative is always positive for all t that the derivative is defined.

b. $f(t)$ is zero when $\sin(2t) = 0$. The sine function is zero when its argument is an integer multiple of π. For $t \in [0, 2\pi]$, $f(t) = 0$ at $t = 0, \pi/2, \pi, 3\pi/2, 2\pi$. The cosine function is zero when its argument is $\pi/2 + n\pi$ for n an integer. Thus, the vertical asymptotes occur halfway between zeroes of f, so at $t = \pi/4, 3\pi/4, 5\pi/4, 7\pi/4$.

c. The graph of $f(t)$ for $t \in [0, 2\pi]$ is below to the left.



Problem 3



Problem 4

4. a. The damped spring-mass system, $y(t) = 2e^{-2t}\sin(2t)$, has a velocity

$$\begin{aligned} v(t) = y'(t) &= 4e^{-2t}\cos(2t) - 4e^{-2t}\sin(2t) \\ &= 4e^{-2t}(\cos(2t) - \sin(2t)) \end{aligned}$$

b. The maximum occurs when $\cos(2t) = \sin(2t)$ or $t = \pi/8$. Thus, the maximum is

$$y(\pi/8) = 2e^{-\pi/4}\sin(\pi/4) \simeq 0.6448.$$

The mass returns to $y(t) = 0$ when $\sin(2t) = \sin(\pi)$ or $t = \pi/2$. Above to the right is a graph of the mass.

5. a. The basilar fiber vibrates through zero when the argument of $\sin(t/2)$ equals $n\pi$ for n an integer. It follows that the zeroes occur when $t = 0, 2\pi, 4\pi$.

b. The velocity is given by

$$\begin{aligned} v(t) = z'(t) &= \frac{15}{2}e^{-t/2}\cos(t/2) - \frac{15}{2}e^{-t/2}\sin(t/2) \\ &= \frac{15}{2}e^{-t/2}(\cos(t/2) - \sin(t/2)) \end{aligned}$$

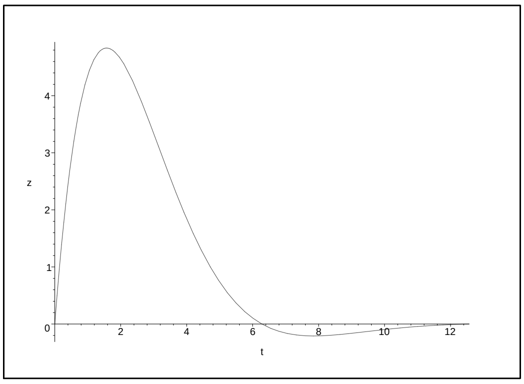
c. The extrema occur when $\cos(t/2) = \sin(t/2)$, so $t/2 = \pi/4 + n\pi$ for n an integer. There is a maximum at $t = \pi/2$ with

$$z(\pi/2) = 15e^{-\pi/4}\sin(\pi/4) \simeq 4.836.$$

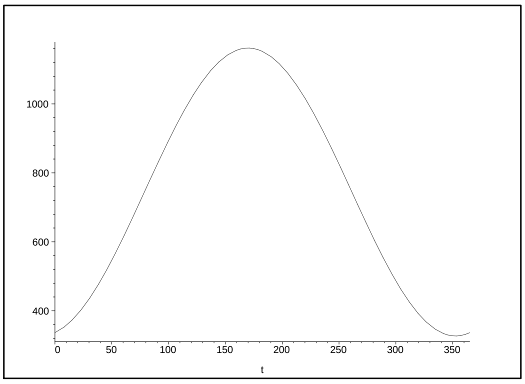
This is followed by a minimum at $t = 5\pi/2$ with

$$z(5\pi/2) = 15e^{-5\pi/4}\sin(5\pi/4) \simeq -0.2090.$$

The graph of $z(t)$ for $t \in [0, 4\pi]$ is shown below to the left.



Problem 5



Problem 6

6. a. The period is 365 days, so $365\omega = 2\pi$ or $\omega = \frac{2\pi}{365} \simeq 0.01721$. The average length of time is $\alpha = \frac{1162+327}{2} = 744.5$ min. The amplitude is given by $\beta = 1162 - 744.5 = 417.5$ min. The maximum occurs on day 170, so $\omega(170 - \phi) = \pi/2$ (based on the maximum of the sine function). Thus, $170 - \phi = \frac{365}{4} = 91.25$ or $\phi = 78.75$ day. It follows that

$$L(t) = 744.5 + 417.5\sin(0.01721(t - 78.75)).$$

The length of day for Ground Hog's day is $L(32) = 744.5 + 417.5\sin(0.01721(32 - 78.75)) = 443.7$ min in Anchorage.

b. The derivative of $L'(t) = 7.185\cos(0.01721(t - 78.75))$. The maximum rate of change occurs when cosine is 1, so $L'(78.75) = 7.185$ min/day, which occurs on day 78.75 or about March 21, the first day of spring. A graph is shown above to the right.

7. a. The differential equation for this culture is $B'(t) = rB(t),\ B(0) = 1000$, which has the solution

$$B(t) = 1000e^{rt}.$$

Since $B(2) = 3000$, we have $1000e^{2r} = 3000$, so $e^{2r} = 3$ or $r = \frac{\ln(3)}{2} \simeq 0.5493$ hr^{-1}. With this value of r, we solve $1000e^{rt_d} = 2000$, so $e^{rt_d} = 2$. Thus, the doubling time is given by $t_d = \frac{\ln(2)}{r} = \frac{2\ln(2)}{\ln(3)} \simeq 1.262$ hr.

b. The mutant population satisfies $M(t) = e^{0.7t}$. Solving $M(t_d) = 2 = e^{0.7t_d}$, we have the doubling time $t_d = \frac{\ln(2)}{0.7} =\simeq 0.9902$ hr. The populations $B(t)$ and $M(t)$ are equal when $1000e^{rt} = e^{0.7t}$ with $r = 0.5493$. So $e^{(0.7-r)t} = 1000$ or

$$t = \frac{\ln(1000)}{0.7 - 0.5493} \simeq 45.84 \text{ hr}.$$

8. a. The solution of the Malthusian growth equation for Japan is $J(t) = 116.8e^{rt}$ (in millions) from the differential equation and the population in 1980. Since the population in 1990 is 123.5 (million), we have $J(10) = 123.5 = 116.8e^{10r}$. Thus,

$$\begin{aligned} e^{10r} &= \frac{123.5}{116.8} \simeq 1.0574 \\ 10r &= \ln(1.0574) \simeq 0.055778 \\ r &= 0.005578 \end{aligned}$$

The doubling time is computed by solving $J(t) = 233.6 = 116.8e^{rt}$, so

$$\begin{aligned} e^{rt} &= 2 \\ rt &= \ln(2) \simeq 0.69315 \\ t &= \frac{\ln(2)}{r} \simeq 124.3 \text{ yr} \end{aligned}$$

b. The differential equation for Bangladesh is given by

$$\frac{dB}{dt} = kB, \quad B(0) = 88.1.$$

A similar calculation is used for Bangladesh, so $B(t) = 88.1e^{kt}$ with $B(10) = 110.1 = 88.1e^{kt}$. Solving for the growth constant k as above, we find

$$k = \frac{1}{10}\ln\left(\frac{110.1}{88.1}\right) \simeq 0.02229.$$

The population in 2000 is found by evaluating

$$B(20) = 88.1e^{20k} \simeq 137.6 \text{ million}.$$

c. The populations of Japan and Bangladesh are equal when $B(t) = J(t)$, so

$$\begin{aligned} 88.1e^{kt} &= 116.8e^{rt} \\ \frac{e^{kt}}{e^{rt}} &= \frac{116.8}{88.1} \simeq 1.3258 \\ e^{(k-r)t} = e^{0.016714t} &= 1.3258 \\ 0.016714t &= \ln(1.3258) \simeq 0.28199 \\ t &= \frac{0.28199}{0.016714} \simeq 16.87 \text{ years} \end{aligned}$$

It follows that these models predict that the population of Bangladesh exceeded the population of Japan in 1997.

9. By examining the initial condition, $P(0) = P_0$, we eliminate the first of the proposed solutions. For (i),

$$P(0) = (P_0 - m)e^0 + \frac{m}{r} = P_0 - m + \frac{m}{r} \neq P_0,$$

for $r \neq 1$. Both of the other solutions are readily seen to satisfy $P(0) = P_0$.

Next we check the proposed solutions (ii) and (iii) in the differential equation. For (ii),

$$\frac{dP}{dt} = (P_0 + m)re^{rt} = r(P_0 + m)e^{rt},$$

while

$$rP+m = r\Big((P_0+m)e^{rt}-m\Big)+m = r(P_0+m)e^{rt}-rm+m.$$

Clearly, $\frac{dP}{dt} \neq rP+m$ (if $r \neq 1$) for this solution.

For (iii),

$$\frac{dP}{dt} = \left(P_0+\frac{m}{r}\right)re^{rt} = r\left(P_0+\frac{m}{r}\right)e^{rt},$$

while

$$\begin{aligned} rP+m &= r\left(\left(P_0+\frac{m}{r}\right)e^{rt}-\frac{m}{r}\right)+m \\ &= r\left(P_0+\frac{m}{r}\right)e^{rt}-m+m \\ &= r\left(P_0+\frac{m}{r}\right)e^{rt}. \end{aligned}$$

Clearly, $\frac{dP}{dt} = rP+m$ for this solution. It follows that (iii) satisfies the initial value problem given by the Malthusian growth model with immigration.

10. a. The solution to the radioactive decay problem is

$$R(t) = 30e^{-kt}.$$

With the half-life of 8 years, $R(8) = 15 = 30e^{-8k}$ or $e^{8k} = 2$. Thus,

$$8k = \ln(2) \qquad \text{or} \qquad k = \frac{\ln(2)}{8} \simeq 0.08664.$$

After 3 days,

$$R(3) = 30e^{-3k} \simeq 23.13 \text{ mCi}.$$

b. The length of time for the original 30 mCi of ^{131}I to decay to 5 mCi of ^{131}I satisfies $R(t) = 5 = 30e^{-kt}$ or $e^{kt} = 6$. It follows that $kt = \ln(6)$ or

$$t = \frac{\ln(6)}{k} \simeq \frac{\ln(6)}{0.08664} \simeq 20.68 \text{ days}.$$

11. a. The solution with the population in millions is given by

$$P(t) = 50.2e^{rt},$$

where t is in years after 1880. From the population in 1890, we have $62.9 = 50.2e^{10r}$ or $e^{10r} = 1.2530$. Thus, $r = 0.02255$. To find the time until the population doubles, we compute $100.4 = 50.2e^{rt}$ or $t = \ln(2)/r \simeq 30.7$. This suggests that the population of the U.S. doubles from 1880 around 1911, assuming that the rate of growth stays constant.

b. The model predicts that the population in 1900 is

$$P(20) = 50.2e^{20r} \simeq 78.8.$$

The error between the model and the actual population is

$$100\frac{(P(20) - 76.0)}{76.0} = 100\frac{(78.8 - 76.0)}{76.0} = 3.7\%.$$

12. a. The solution to the white lead problem is $P(t) = 10e^{-kt}$, where $t = 0$ represents 1970. From the data at 1975, we have $8.5 = 10e^{-5k}$ or $e^{5k} = 10/8.5 = 1.17647$. Thus, $k = 0.032504$ yr^{-1}. To find the half-life, we compute $5 = 10e^{-kt}$, so $t = \ln(2)/k = 21.33$ yr is the half-life of lead-210.

b. The differential equation can be written $P' = -k(P - r/k)$, so we make the substitution $z(t) = P(t) - r/k$. This leaves the initial value problem

$$z' = -kz, \quad z(0) = P(0) - r/k = 10 - r/k,$$

which has the solution $z(t) = (P(0) - r/k)e^{-kt} = P(t) - r/k$. Thus, the solution is

$$P(t) = \left(10 - \frac{r}{k}\right) e^{-kt} + \frac{r}{k} = 2.3086e^{-kt} + 7.6914,$$

where $k = 0.032504$. In the limit,

$$\lim_{t\to\infty} P(t) = 7.6914 \text{ disintegrations per minute of } ^{210}\text{Pb}.$$

13. a. The differential equation describing the temperature of the tea satisfies

$$H' = -k(H - 21), \quad H(0) = 85 \text{ and } H(5) = 81.$$

Make the substitution $z(t) = H(t) - 21$, which gives the differential equation

$$z' = -kz, \quad z(0) = H(0) - 21 = 64.$$

The solution becomes $z(t) = 64e^{-kt} = H(t) - 21$ or

$$H(t) = 64e^{-kt} + 21.$$

To find k, we solve $H(5) = 81 = 64e^{-5k} + 21$ or $e^{5k} = 64/60 = 1.0667$. Thus, $k = 0.012908$ min^{-1}. The water was at boiling point when $64e^{-kt} + 21 = 100$ or $e^{-kt} = 79/64$. It follows that $t = -\ln(79/64)/k = -16.3$ min. This means that the talk went 16.3 min over its scheduled ending.

b. To obtain a temperature of at least 93°C, then we need to find the time that satisfies $H(t) = 93 = 64e^{-kt} + 21$, so $e^{-kt} = 72/64 = 1.125$. Solving for t gives $t = -\ln(72/64)/k = -9.125$ min. It follows that you must arrive at the hot water within $16.3 - 9.1 = 7.2$ min of the scheduled end of the talks.

14. a. Substituting the parameters into the differential equation gives

$$c' = \frac{1}{10^6}(22000 - 2000c) = -0.002(c - 11).$$

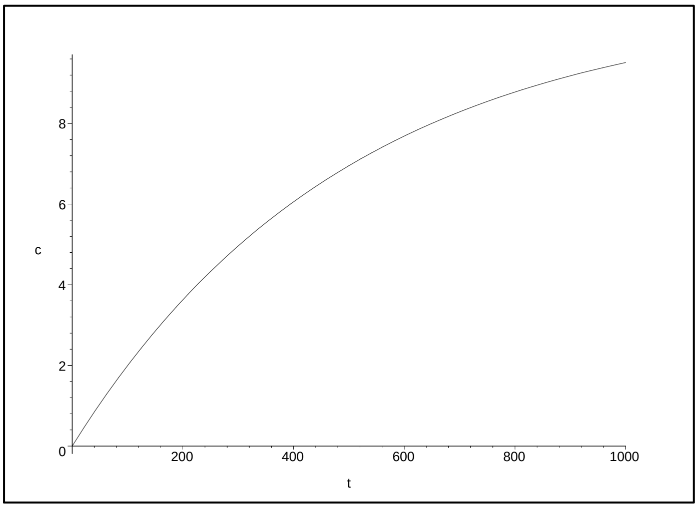
We make the substitution $z(t) = c(t) - 11$, which gives the initial value problem $z' = -0.002z$ with $z(0) = c(0) - 11 = -11$. The solution of this differential equation is $z(t) = -11e^{-0.002t} = c(t) - 11$, so

$$c(t) = 11 - 11e^{-0.002t}.$$

b. Solve the equation $c(t) = 11 - 11e^{-0.002t} = 5$, so $e^{0.002t} = 11/6$ or $t = 500\ln(11/6) = 303.1$ days. The limiting concentration

$$\lim_{t\to\infty} c(t) = 11.$$

The graph is below.



15. a. Substituting the parameters into the differential equation gives

$$\frac{dc(t)}{dt} = \frac{200}{10000}(10 - c) = -0.02(c - 10).$$

We make the substitution $z(t) = c(t) - 10$, which gives the initial value problem $z' = -0.02z$ with $z(0) = c(0) - 10 = -10$. The solution of this differential equation is $z(t) = -10e^{-0.02t} = c(t) - 10$, so

$$c(t) = 10 - 10e^{-0.02t}.$$

We solve the equation $c(t) = 10 - 10e^{-0.02t} = 2$, so $e^{0.02t} = 10/8$ or $t = 50\ln(10/8) = 11.2$ days.

b. The Euler's formula with the linearly increasing pollutant level is given by

$$c_{n+1} = c_n + h(0.02(10 + 0.1t_n - c_n)) = c_n + 0.2 + 0.002t_n - 0.02c_n,$$

with $h = 1$. Iterating this, we create a table

$t_0 = 0$	$c_0 = 0$
$t_1 = 1$	$c_1 = c_0 + 0.2 + 0.002t_0 - 0.02c_0 = 0.2$
$t_2 = 2$	$c_2 = c_1 + 0.2 + 0.002t_1 - 0.02c_1 = 0.398$

The approximate solution is $c_2 = 0.398$ ppb.

16. a. For the differential equation $\dfrac{dy}{dt} = t(2-y)$, the Euler formula is given by

$$y_{n+1} = y_n + h\left(t_n(2-y_n)\right) = y_n + 0.25\left(t_n(2-y_n)\right).$$

For this problem, $y_0 = 4$, we can use the Euler's formula to create the following table:

$t_0 = 0$	$y_0 = 4$
$t_1 = 0.25$	$y_1 = y_0 + 0.25\left(t_0(2-y_0)\right) = 4 + 0.25(0)(2-4) = 4$
$t_2 = 0.5$	$y_2 = y_1 + 0.25\left(t_1(2-y_1)\right) = 4 + 0.25(0.25)(2-4) = 3.875$
$t_3 = 0.75$	$y_3 = y_2 + 0.25\left(t_2(2-y_2)\right) = 3.875 + 0.25(0.5)(2-3.875) = 3.6406$
$t_4 = 1.0$	$y_4 = y_3 + 0.25\left(t_3(2-y_3)\right) = 3.6406 + 0.25(0.75)(2-3.6406) = 3.3330$

Thus, the approximate the solution at $t = 1$ is $y_4 \simeq y(1) = 3.3330$.

b. First, we note that all of the choices satisfy the initial condition. If we differentiate the solutions from the choices given we have:

$$\begin{array}{ll} (i) & y'(t) = -2e^{-t} \\ (ii) & y'(t) = 2t \\ (iii) & y'(t) = -2te^{-\frac{t^2}{2}} \end{array}$$

The right hand side of the differential equation is $t(2-y)$, so we have

$$\begin{array}{ll} (i) & t(2-y) = t(2e^{-t}) \\ (ii) & t(2-y) = t(t^2+2) \\ (iii) & t(2-y) = t(-2e^{-\frac{t^2}{2}}) \end{array}$$

Only the last choice satisfies the differential equation. We evaluate $y(1) = 2 + 2\,e^{-1/2} = 3.21306$, which gives an error of

$$100\frac{y_4 - y(1)}{y(1)} = 100\frac{3.3330 - 3.21306}{3.21306} = 3.73\%.$$

17. a. For the differential equation, $\frac{dy}{dt} = y + 2$ with $y(0) = 3$ and $h = 0.25$, the Euler's formula is

$$y_{n+1} = y_n + h(y_n + 2) = y_n + 0.25(y_n + 2).$$

Iterating this, we create a table

$t_0 = 0$	$y_0 = 3$
$t_1 = 0.25$	$y_1 = y_0 + 0.25(y_0 + 2) = 3 + 0.25(3+2) = 4.25$
$t_2 = 0.5$	$y_2 = 4.25 + 0.25(4.25 + 2) = 5.8125$
$t_3 = 0.75$	$y_3 = 4.25 + 0.25(4.25 + 2) = 7.7656$
$t_4 = 1.0$	$y_4 = 7.7656 + 0.25(7.7656 + 2) = 10.2070$

Thus, the approximate the solution at $t = 1$ is $y_4 \simeq y(1) = 10.2070$.

b. The differential equation is a linear differential equation, so we make a substitution, $z(t) = y(t) + 2$. The solution to the initial value problem is

$$z' = z, \quad z(0) = y(0) + 2 = 5.$$

This has the solution $z(t) = 5e^t = y(t)+2$, so $y(t) = 5e^t - 2$. It follows that $y(1) = 5e - 2 \simeq 11.5914$. The error between the actual and Euler's solution is

$$100\frac{(y_4 - y(1))}{y(1)} = 100\frac{(10.2070 - 11.5914)}{11.5914} = -11.94\%,$$

which implies that Euler's method falls significantly short of the actual solution.

18. a. The solution to this differential equation is $R(t) = 10e^{-0.05t}$. The half-life satisfies $5 = 10e^{-0.05t}$, so $e^{0.05t} = 2$ or $t = 20\ln(2) \simeq 13.86$.

b. For the differential equation, $\frac{dR}{dt} = -0.05R + 0.2e^{-0.01t}$ with $R(0) = 10$ and $h = 1$, the Euler's formula is

$$R_{n+1} = R_n + h(-0.05R_n + 0.2e^{-0.01t_n}) = R_n - 0.05R_n + 0.2e^{-0.01t_n}.$$

Iterating this, we create a table

$t_0 = 0$	$R_0 = 10$
$t_1 = 1$	$R_1 = R_0 - 0.05R_0 + 0.2e^{-0.01t_0} = 10 - 0.5 + 0.2 = 9.7$
$t_2 = 2$	$R_2 = R_1 - 0.05R_1 + 0.2e^{-0.01t_1} = 9.7 - 0.485 + 0.198 = 9.413$
$t_3 = 3$	$R_3 = R_2 - 0.05R_2 + 0.2e^{-0.01t_2} = 9.413 - 0.471 + 0.096 = 9.138$

Thus, the approximate the solution at $t = 3$ is $R_3 \simeq R(3) = 9.138$.

c. Consider $R(t) = 5e^{-0.05t} + 5e^{-0.01t}$, then $R'(t) = -0.25e^{-0.05t} - 0.05e^{-0.01t}$. But $-0.05R(t) + 0.2e^{-0.01t} = -0.05(5e^{-0.05t} + 5e^{-0.01t}) + 0.2e^{-0.01t} = -0.25e^{-0.05t} - 0.05e^{-0.01t}$, so the second choice (ii) is the solution to the radioactive decay problem with another input. The correct solution at $t = 3$ is $R(3) = 9.15576$. The percent error between the correct solution and the Euler solution

$$100\frac{(R(3) - R_3)}{R(3)} = 100\frac{(9.15576 - 9.138)}{9.15576} = 0.19\%.$$

19. a. The differential equation with the information in the problem is given by:

$$\frac{dH}{dt} = -k(H - 25), \qquad H(0) = 35,$$

where $t = 0$ is 7 AM. We make the change of variables $z(t) = H(t) - 25$, so $z(0) = 10$. The problem now becomes

$$\frac{dz}{dt} = -kz, \qquad z(0) = 10,$$

which has the solution

$$z(t) = 10\,e^{-kt} \quad \text{or} \quad H(t) = 25 + 10\,e^{-kt}.$$

From the information at 9 AM, we see

$$H(2) = 33.5 = 25 + 10\,e^{-2k} \quad \text{or} \quad e^{2k} = \frac{10}{8.5} \quad \text{or} \quad k = \frac{\ln\left(\frac{10}{8.5}\right)}{2} = 0.081259.$$

It follows that

$$H(t) = 25 + 10\,e^{-0.081259t}.$$

The time of death is found by solving

$$H(t_d) = 39 = 25 + 10\,e^{-0.081259t_d} \quad \text{or} \quad e^{-0.081259t_d} = \frac{14}{10} \quad \text{or} \quad t_d = -\frac{\ln(1.4)}{0.081259} = -4.1407.$$

It follows that the time of death is 4 hours and 8.4 min before the body is found, which gives the time of death around 2:52 AM.

20. a. Let $A(t)$ be the amount of drug in the body, then the concentration of the drug is given by $c(t) = A(t)/10$. We first write the differential equation for the change in amount of drug in the body

$$\frac{dA}{dt} = \mathit{amt\ entering} - \mathit{amt\ leaving} = 1(0.2) - 1 \cdot c.$$

The differential equation for the concentration of drug satisfies

$$\frac{dc}{dt} = 0.02 - 0.1c = -0.1(c - 0.2), \qquad c(0) = 0.$$

Let $z(t) = c(t) - 0.2$, then we transform the linear differential equation above into

$$\frac{dz}{dt} = -0.1z, \qquad z(0) = -0.2.$$

which has the solution

$$z(t) = -0.2\,e^{-0.1t} \quad \text{or} \quad c(t) = 0.2 - 0.2\,e^{-0.1t}.$$

b. The tumor responds when $c(t) = 0.1$, solving $c(t) = 0.1 = 0.2 - 0.2\,e^{-0.1t}$ or $e^{0.1t} = 2$. It easily follows that the time for a response to begin is $t = 10\ln(2) = 6.9315$ days.

c. If the body metabolizes 0.05 μg/day, then the new equation for the amount of drug in the body is

$$\frac{dA}{dt} = 1(0.2) - 0.05 - 1 \cdot c = 0.15 - c.$$

The differential equation for the concentration of drug satisfies

$$\frac{dc}{dt} = 0.015 - 0.1c = -0.1(c - 0.15), \qquad c(0) = 0.$$

The limiting concentration is reached when $\frac{dc}{dt} = 0$. Substituting this in the differential equation above, we see

$$0 = -0.1(c - 0.15) \quad \text{or} \quad c = 0.15.$$

It follows that

$$\lim_{t\to\infty} c(t) = 0.15\ \mu\text{g/l}.$$