Give all answers to at least **4 significant figures**.

1. a. It can be shown that puppies put on weight (in kg) in a manner that fits the function

$$W(t) = \frac{W_0 M e^{rt}}{M + W_0(e^{rt} - 1)},$$

where W_0 represents the birth weight of the puppy, M is the final weight, r is a growth rate, W is the weight in kg, and t is the age in days. Below is a table for the growth of a Golden Retriever puppy.

t (day)	Weight (kg)
0	1.5
20	2.5
40	4.1
60	6.8
100	13.6
150	24.5
200	29.5
250	32.0
350	34.2
450	35

a. Use Excel to find the best values of the parameters W_0, M, and r. Write the values of these parameters and the formula for the model that best fits the weight of the puppy in the study above. What does this model predict for the weight of the puppy at 1 and 5 years of age?

b. The ASPCA Dog Care Manual gives the following table for the amount of calories that a dog needs for a given weight:

Weight (kg)	Calories
5	450
10	750
20	1250
30	1700
40	2100
50	2500

Use Excel's Trendline to find the best allometric model of the form

$$C(W) = kW^a,$$

where k and a are constants that fits the data above. Write the values of these constants and the equation for the model.

c. Create a composite function to give the amount of calories needed as a function of age from the two models above, *i.e.*, find $C(t)$. Write this function and its derivative, $C'(t)$.

d. Find the point of inflection, which is when the rate of change in the amount of calories is at a maximum. Give the age, t, when this occurs, as well as the number of calories required $C(t)$ and rate of change in the amount of calories $C'(t)$ at this time.

2. In 1946, A. C. Crombie studied a number of populations of insects with the amount of food supplied strictly regulated. One study examined *Rhizopertha dominica*, the lesser grain borer. A slightly modified set of his population data is given in the table below:

Week	Population
0	2
2	2
4	3
6	17
8	65
10	119
12	130
14	175
16	205
18	261
20	302
22	330
24	315
26	333
28	350
30	332

In this problem, we want to compare the discrete logistic growth and Beverton-Holt models. For an adult population, P_n, the discrete logistic growth model is given by:

$$P_{n+1} = f(P_n) = rP_n - mP_n^2,$$

where the constants r and m are determined from the data. Similarly, the Beverton-Holt model is given by:

$$P_{n+1} = B(P_n) = \frac{aP_n}{1 + bP_n},$$

where the constants a and b are determined from the data. This problem is similar to the beetle problem in Lab 1, where you find the updating function, then use the updating function to simulate the time series.

a. Begin this problem by finding the **two updating functions**, $f(P_n)$ and $B(P_n)$, given above. Plot P_{n+1} vs. P_n, which you can do by entering the adult population data from times 0-28 for P_n and times 2-30 for P_{n+1}. (Be sure that P_n is on the horizontal axis.) For the logistic growth function, $f(P_n)$, use Excel's trendline with its polynomial fit of order 2 and with the intercept set to 0 (under options). Write the best constants, r and m, and determine the sum of square errors between the data and the updating function.

You continue by finding the best fitting Beverton-Holt updating function, $B(P_n)$. For this function, we find the least squares best fit of this function to the data. The initial guess of a should be equal to the value of r found above, while the initial guess for b is r/M, where M is the highest population in the data. From the sum of square errors, use Excel's Solver to find the best fitting parameters a and b and give the sum of square errors. Compare this number to the one for the logistic growth updating function. Which updating function fits the data better.

b. Find the equilibria for both the logistic growth and Beverton-Holt models using the best fitting parameters found above. Write the derivative of both updating functions. Evaluate the derivatives at each of the equilibria, then discuss the behavior (stability) of these models near their equilibria.

c. Take each of these models with an initial population that is to be determined by the sum of square errors between the time series data and the simulated models. (Two simulations, one for the logistic growth model and one for the Beverton-Holt model.) Use Solver to find the value of the best initial conditions for each of the models. (Modify only the initial condition for each of the models, using the parameters found above for the updating functions in the model.) List the simulated values at times 10, 20, and 30 weeks for each of the models. Write the sum of square errors between the models with the best initial population. Which model fits the data better? Is there a significant difference between these two models. Discuss how well your simulation matches the data in the table.

[1] A. C. Crombie (1946) On competition between different species of graminivorous insects, *Proc. R. Soc. (B)*, **132**, 362-395.