1. Consider the trigonometric function:

$$y = 2 - 4\cos(2x), \qquad x \in [0, 2\pi].$$

Find the period of this function. Give the x and y values for all maxima in the specified interval. Sketch the graph of this function.

2. Consider the trigonometric function:

$$y = 5\sin(3x) - 4, \qquad x \in [0, 2\pi].$$

Find the period of this function. Give the x and y values for all maxima in the specified interval. Sketch the graph of this function.

3. Consider the trigonometric function:

$$y = 7 - 4\cos\left(\frac{\pi}{8}(t-5)\right), \qquad t \in [0, 20].$$

Find the period, amplitude, phase shift, and vertical shift of this function. Give the t and y values for all absolute maxima $(t_{max}, y(t_{max}))$ and absolute minima $(t_{min}, y(t_{min}))$ in the specified interval. (Note that there could be more than one maximum or minimum.) Sketch the graph of this function.

4. Consider the Logistic growth model given by the discrete dynamical model

$$P_{n+1} = F(P_n) = 2.8P_n - 0.0005P_n^2,$$

where P_n is the population after n generations.

a. Suppose that initially there are 1000 individuals, so $P_0 = 1000$. Find the populations at the end of the first two generations P_1 and P_2.

b. Find all equilibria for this model, then use the derivative of the updating function, $F'(P)$, to determine the behavior of the solution near the equilibria.

c. Sketch the updating function and the identity function ($P_{n+1} = P_n$), showing the vertex of $F(P)$, the points of intersection, and any intercepts.

5. Certain gregarious animals require a minimum number of animals in a colony before they reproduce successfully. This is called the *Allee effect*. Consider the following model for the population of a gregarious bird species, where the population, N_n, is given in thousands of birds:

$$N_{n+1} = N_n + 0.1N_n\left(1 - \frac{1}{9}(N_n - 5)^2\right).$$

a. Assume that the initial population is $N_0 = 4$, then determine the population for the next two generations (N_1 and N_2).

b. Find all equilibria for this model.

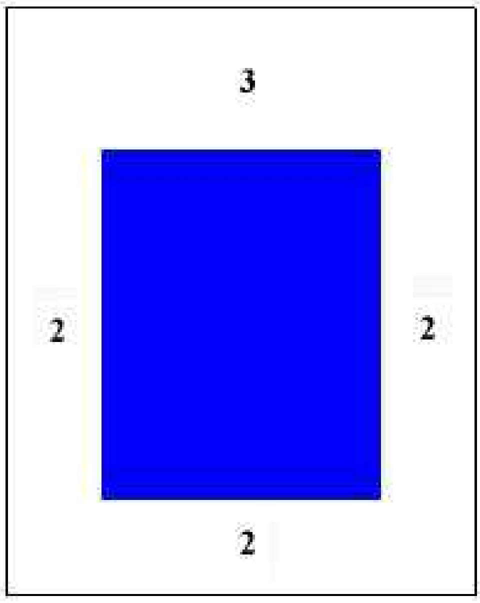
c. The model above can be expanded to give

$$N_{n+1} = A(N_n) = \frac{37}{45}N_n + \frac{1}{9}N_n^2 - \frac{1}{90}N_n^3.$$

Find the derivative of $A(N)$. Evaluate the derivative $A'(N)$ at each of the equilibria found above and determine the local behavior of the solution near each of those equilibria.

d. Give a brief biological description of what your results imply about this gregarious species of bird.

6. A brochure is to have an area of 125 in^2, with a 3 in margin at the top and 2 in margins on the sides and bottom. Find the dimensions of the brochure that allow the maximum printing area. (See figure below.)



7. An open box with a square base is to be constructed with 600 cm^2 of material. Find its dimensions that maximize the volume.

8. Suppose that a study finds that the best fit for the number of drops required to break open a walnut satisfies the equation

$$N(h) = 1 + \frac{10}{h-1},$$

where h is the height of the drop in meters. The energy used to break open a walnut is given by the function

$$E(h) = hN(h).$$

Find the height that a crow should fly to minimize the energy needed to break open a walnut.

9. Let P_n be the population of fish after n months and assume that their population dynamics can be approximated by Ricker's growth model

$$P_{n+1} = R(P_n) = 6P_n e^{-0.001P_n}.$$

a. Assume that the initial population is $P_0 = 100$, then determine the population for the next two generations (P_1 and P_2).

b. From the model above, we see that the growth function is given by

$$R(P) = 6Pe^{-0.001P}.$$

Find $R'(P)$, then determine the maximum of this function (both P and $R(P)$ values). Evaluate the $\lim_{P\to\infty} R(P)$. Give any intercepts and asymptotes for this function. Also find any points of inflection. Sketch its graph for $P \geq 0$.

c. Find all equilibria for Ricker's model and determine the stability of the equilibria. (Give the numerical value of the derivative at the equilibria to justify your stability argument.)

10. Hassell's model is often used for the dynamics of insect populations with discrete generations. Suppose that a population of insects satisfies the discrete model

$$P_{n+1} = H(P_n) = \frac{16P_n}{(1+0.005P_n)^2}.$$

a. Suppose $P_0 = 500$ and find the populations in the next two generations, P_1 and P_2.

b. The updating function is

$$H(P) = \frac{16P}{(1+0.005P)^2}.$$

Find $H'(P)$, then determine the maximum of this function (both P and $H(P)$ values). Evaluate the $\lim_{P\to\infty} H(P)$. Give any intercepts and asymptotes for this function. Also find any points of inflection. Sketch its graph for $P \geq 0$.

c. Find all equilibria for the model above and determine the stability of those equilibria. (Give the numerical value of the derivative at the equilibria to justify your stability argument.)

11. A typical nerve action potential is characterized by a sharp outflowing of sodium ions leading to a depolarization of the nerve membrane. Next potassium ions flow inward causing a repolarization. There is a period of time afterwards that the membrane is hyperpolarized, which prevents further stimulation of the nerve before returning to resting potential. A cubic equation that approximates the data for a typical action potential of a nerve cell is given by

$$V(t) = 50t(t-2)(t-3) - 70,$$

where t is the time in msec following stimulation of the nerve cell and V is membrane potential in mV.

a. The membrane potential is considered at rest when $V(t) = -70$ mV. Find the times when this model predicts the membrane is at rest.

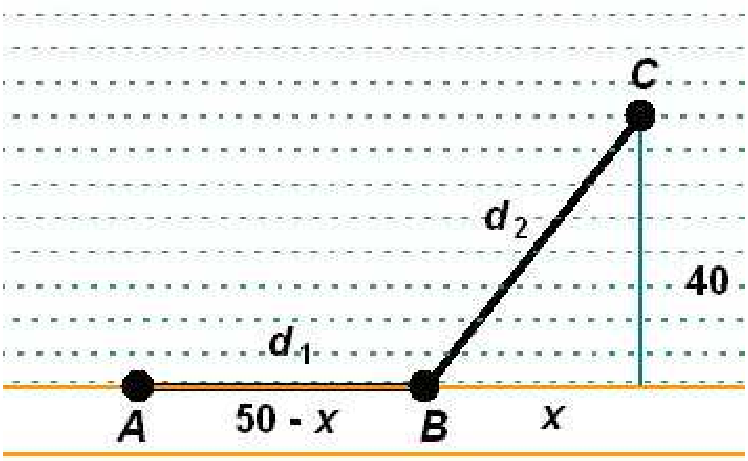
b. Find the time and potential of the membrane at the peak of the action potential. Also, find the time and membrane potential of the membrane when it is most hyperpolarized (minimum potential). Sketch a graph of this function for $0 \le t \le 3$.

12. A rabbit is being chased by a predator and wants to reach its burrow as quickly as possible. (See the diagram below.) Assume that it is initially at Point A with the burrow residing at Point C. Assume it can run along the road at $v_1 = 15$ m/sec and through the brush at $v_2 = 9$ m/sec. The burrow is 40 m from the road, and the rabbit is 50 m up the road from the closest point of the burrow to the road. If the distance it runs along the road (from A to B) is d_1 and the distance it runs in the brush (from B to C) is d_2, then the time for the rabbit to reach the burrow is given by the formula

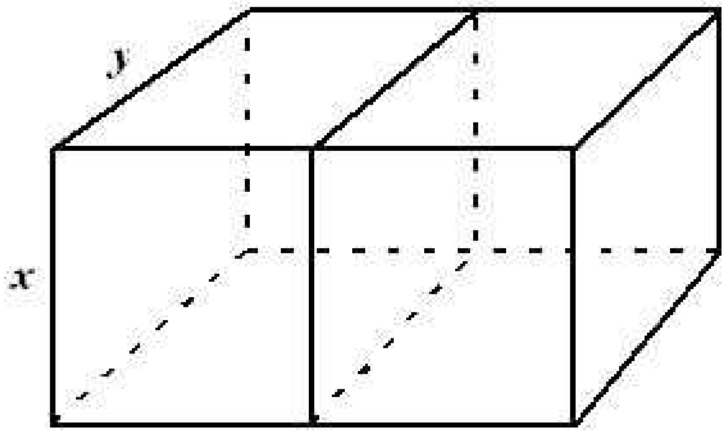
$$T = \frac{d_1}{v_1} + \frac{d_2}{v_2}.$$

a. Use the diagram below to form an expression for the time as a function of x (the distance down the road to where the rabbit enters the brush), $T(x)$.

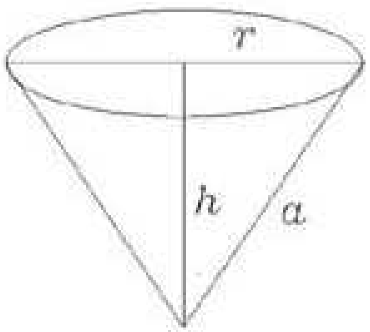
b. Use your expression for the time $T(x)$ to find the minimum time for the rabbit to reach its burrow. Give both the distance x and the time at the minimum.

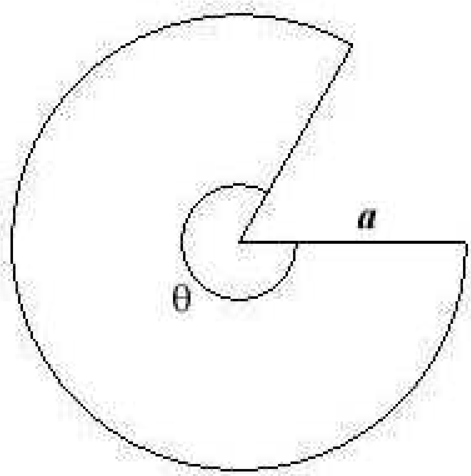


13. A student in ecology needs to design two experimental holding pens for their species of animal. Below is a design of the two pens that are each square on their ends. The animal needs 50,000 cm^3 of space in each of these holding pens. (Note that the interior side is shared by both pens, so be **very** careful when you count the sides.) Special material is used for the sides of the pens and back, while the front end screened with a door (so is not included in the calculations). Find the dimensions width and depth (x and y) for each of the holding pens that minimizes the surface area of the enclosed space.



14. A water cup in the shape of a right circular cone is to be constructed by removing a sector from a circular sheet of paper of radius a and then joining the two straight edges of the remaining paper. Find the dimensions of the cup with the largest volume that can be constructed in this way. (Hint: From the diagrams below, the volume of the cone is $V = \pi r^2 h/3$, where the variables r and h satisfy $r^2 + h^2 = a^2$. Furthermore, the circumference of the base of the cone is equal to the length of the sector of the circle, so $2\pi r = a\theta$. Use this information to create a function for the volume of the cone that depends only on θ, then use standard optimization methods to find the maximum volume.)





15. The growth in length of a leopard shark is approximated by the von Bertalanffy equation

$$L(t) = 1.6(1 - 0.85e^{-0.08t}),$$

where t is in years and L is in m. (Sharks often start life fairly large and mature slowly.) An allometric measurement of a leopard shark shows that its weight can be approximated by the model

$$W(L) = 4.5L^3,$$

where W is in kg.

a. Find the intercepts ($t \geq 0$) and any asymptotes for the length of a leopard shark, then sketch of graph showing the length of a leopard shark as it ages. If it reaches sexual maturity at a length of 0.5 m, then at what age is a leopard shark sexually mature.

b. Create a composite function to give the weight of a leopard shark as a function of its age, $W(t)$. Find the intercepts and any asymptotes for $W(t)$, then sketch of graph showing the weight of a leopard shark as it ages.

c. Find the derivative of $W(t)$ using the chain rule. Also, compute the second derivative, then determine when this second derivative is zero. From this information, find at what age leopard sharks are increasing their weight the most and determine what that weight gain is. Be sure to give the units of weight gain.

16. The muscles of the small intestine move chyme (food and enzymes) toward the colon in a process called *peristalsis* at a rate of 1-10 cm/sec. They periodically contract to create a traveling wave that causes the fluid in the small intestine to flow forward and allow absorption of nutrients into the blood. Consider a cross-section of the small intestine, assuming that it maintains a circular shape with a radius of $R(t)$ under the smooth muscle contraction. Suppose that the radius of one segment of the small intestine satisfies the function

$$R(t) = A + B\cos(\omega t),$$

where you must find the constants A, B, and ω.

a. While digesting food, assume that the segment of small intestine periodically contracts 10 times per minute. Assume that the maximum distention of this segment occurs at $t = 0$ and is 4 cm ($R(0) = 4$). When the muscle is maximally contracted, the opening of the segment in the intestine reduces to only a radius of 1 cm (the minimum of $R(t)$). With these data, find the constants A, B, and ω.

b. Sketch the graph of $R(t)$ for $t \in [0, 0.2]$. List all maxima and minima in the interval.

17. In lab we considered a model for the length of day in San Diego. The effect is dramatically more pronounced when you go to Alaska. In Anchorage, Alaska, the longest day is June 20 at 19 hr 22 min or 1162 min. The shortest day is December 21 at 5 hr 27 min or 327 min. Consider a model for the length of the day in minutes, $L(t)$, as a function of the date, t, using the sine function as follows

$$L(t) = \alpha + \beta\sin(\omega(t - \phi)),$$

where the constants α, β, ω, and ϕ are to be determined below (and assuming that January 1 is $t = 0$).

Assume that June 20 is given by day 170 with length of 1162 min. With the information that the shortest day is 327 min and a year is 365 days, find the constants α, β, ω, and ϕ. Write the function $L(t)$ and find the length of Ground Hog day (February 2 or Day 32) in Anchorage.

18. a. The population of Mexico in 1950 was about 28.49 million, while in 1980, it was about 68.34 million. Assume that the population is growing according to the discrete Malthusian growth equation

$$P_{n+1} = (1 + r)P_n, \quad \text{with} \quad P_0 = 28.49,$$

where P_0 is the population in 1950 and n is in decades. Use the population in 1980 (P_3) to find the value of r (to **4 significant figures**). Find how long it would take for this population to double.

b. Estimate the population in 2000 based on the Malthusian growth model. Given that the population in 2000 was 99.93 million, find the percent error between the actual and predicted values.

c. A better model fitting the census data for Mexico is a logistic growth model given by

$$P_{n+1} = F(P_n) = 1.48P_n - 0.0035P_n^2,$$

where again n is in decades after 1950. Estimate the populations in 1960 and 1970 by computing P_1 and P_2, where $P_0 = 28.49$.

d. Find the equilibrium for this logistic growth model. Calculate the derivative of $F(P)$ and evaluate it at the larger of the equilibria. What does this value say about the behavior of the solution near this equilibrium? Is it stable or unstable? Is it monotonic or oscillatory?

19. Rattlesnakes are cold-blooded or ectothermic organisms with their body temperature depending on the external temperature. The body temperature of a rattlesnake tracks the external temperature with variations. Suppose that the low temperature recorded for the rattlesnake on one day is 10°C at about 4 am, while the high is 26°C at about 4 pm. Assume that the body temperature for the rattlesnake can be modeled using the following function:

$$T(t) = A - B\sin(\omega(t - \phi)),$$

where A, B, ω, and ϕ are constants and t is in hours. Use the data above to find the four parameters, then sketch a graph for the temperature of the rattlesnake for this day (midnight to midnight).