

1. $P(t) = 1,000,000e^{0.03t}$. Doubling time, $t_d = \frac{100}{3} \ln(2) \simeq 23.1$ min. $P(60) = 6,049,647$.

2. Rate constant $r = \frac{1}{7} \ln(2) \simeq 0.099$ yr⁻¹. $P(20) = 3623$.

3. Rate constant $k = \frac{1}{30} \ln(2) \simeq 0.0231$ day⁻¹. $R(10) = 3.97$ mg.

4.

a. $y(t) = ce^{-t} + 1$
b. $y(t) = ce^{-t}$
c. $y(t) = t - t^2 + c$
d. $y(t) = ce^{2t}$
e. $y(t) = ce^{t^2}$

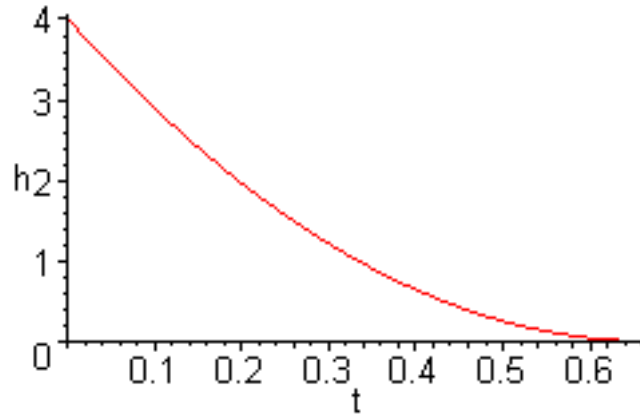
5.

a. $y(t) = c \cos(2t), \quad c \sin(2t)$
b. $y(t) = ce^{-t} \cos(t), \quad ce^{-t} \sin(t)$
c. $y(t) = ce^{2t}, \quad ce^{-2t}$
d. $y(t) = ce^{2t}, \quad ce^{-t}$

6. Differentiating, we see

$$h'(t) = 2(2 - 3t)(-3) = -6(2 - 3t) = -6\sqrt{(2 - 3t)^2} = -6\sqrt{h(t)}.$$

At $t = 0$, $h(0) = 2^2 = 4$. The bucket empties at $t = \frac{2}{3}$ hr. The graph is below:



7. At $t = 0$, $p(0) = 1000(1 + 9)^{-1} = 100$. Differentiating $p(t)$, we see

$$p'(t) = -1000(1 + 9e^{-0.1t})^{-2}(-0.9e^{-0.1t}) = \frac{900e^{-0.1t}}{(1 + 9e^{-0.1t})^2}.$$

Substituting $p(t)$ into the right hand side of the equation:

$$0.1p \left(1 - \frac{p}{1000}\right) = \frac{100}{1 + 9e^{-0.1t}} \left(1 - \frac{1}{1 + 9e^{-0.1t}}\right) = \frac{100}{1 + 9e^{-0.1t}} \left(\frac{9e^{-0.1t}}{1 + 9e^{-0.1t}}\right).$$

Thus, the differential equation is satisfied. The limit satisfies

$$\lim_{t \rightarrow \infty} p(t) = 1000.$$

The graph of the solution is below.

