

Name: _____

Section (1 pt): 8AM, 5PM, 6PM, 7PM, MAP

Perm # (1 pt): _____

MATH 3C MIDTERM
Lecture 2, W'03, Feb 10, 2003

All your answers must be carefully justified. It is not enough to have a correct answer, you must explain how you got it. Neat work, clear and to-the-point explanations will receive more credit than messy, chaotic answers. You may not use books, notes, and calculators on this exam. You may have a 3'' x 5'' handwritten cheat sheet.

Simplify your answers as much as you can. It's ok to leave $\sqrt{3}$, π and such, but not 7/14. Put your answers on this sheet. You may use the back of the sheets if you need more space. This exam has 2 pages.

1. (2 pts for each correct answer, -1 pt for each wrong answer, 0 for blank) Decide if the following statements are true (i.e. always true) or false (i.e. false in at least one case). You need not justify your answer.

0	2
1	
2	
3	
4	
Σ	

For office use only.
Don't put anything
in these boxes.

- (a) The level curves of the function $f(x, y)$ are the isoclines of the DE $y' = f(x, y)$.
- (b) If L is a linear operator and y is a differentiable function, then $L(cy) = cL(y)$.
- (c) If L is an operator and $cL(y) = L(cy)$ for any differentiable function y , then L is a linear operator.
- (d) The DE $y' - (x^2 - x)(y^2 - 1) = 0$ has no equilibrium solutions.
- (e) The DE $y' + x^2y - 1 = 0$ is autonomous.
- (f) Not every autonomous first-order DE is separable.
- (g) The DE $\frac{y'}{y} - e^xy = 0$ is linear.
- (h) The DE $y'' - \ln\left(\frac{x}{e^y}\right) = 0$ is a linear and homogeneous.
- (i) If y is an equilibrium solution of a first-order DE, then $\frac{dy}{dx} = 0$.
- (j) Let L be a linear differential operator and y_1 and y_2 two solutions of the DE $L(y) = e^{-x^2}$. Then $y_1 - y_2$ is also a solution of the same DE.

2. (10 pts) Find the equation of the tangent plane to the function $f(x, y) = y^x - \ln(1/x^2)$ at the point (2, 3).

3. (10 pts) In this problem, you will use Euler's method with a stepsize of $\Delta x = 0.5$ to approximate the solution of the IVP

$$\frac{dy}{dx} = y + 2x, \quad y(1) = -1.$$

Fill out the table below:

x	y	$y + 2x$	Δy
1			
1.5			
2			

4. (15 pts) In this problem, you will find solutions of the DE

$$\frac{dy}{dx} = \frac{(2y^3 - 3y)2x \ln x}{6y^2 - 3} \quad (x > 0).$$

(a) Find the equilibrium solutions if there are any.

(b) Use separation of variables to find all the nonequilibrium solutions. You may not be able to solve for y explicitly.

(c) Check your solution by differentiating it. (Hint: you may have to use implicit differentiation.)

(d) Find the particular solution that fits the initial condition $y(1) = -1$.